

B SHAPE AND SPACE

REVIEW #2

Solution Key

Transformations  
— Transformations

Clarification: Students need to be familiar with the term “invariant points” as points that are not altered by a transformation.

B1 describe how vertical and horizontal translations of functions affect graphs and their related equations:

$y = f(x - h)$

$y - k = f(x)$

Knowledge

B1

1. If the graph of  $2x + 3y = 5$  is translated 4 units up, determine an equation of the new graph.

- A.  $2x + 3y = 1$
- B.  $2x + 3y = 9$
- C.  $2x + 3(y + 4) = 5$
- D.  $2x + 3(y - 4) = 5$

$y \rightarrow y - 4$   
 $2x + 3(y - 4) = 5$

Understanding

B1

2. If  $(a, b)$  is a point on the graph of  $y = f(x - 2) + 3$ .

- A.  $(a - 2, b + 3)$
- B.  $(a - 2, b - 3)$
- C.  $(a + 2, b + 3)$
- D.  $(a + 2, b - 3)$

$x \rightarrow x - 2$     2 right  
 $y \rightarrow y - 3$     3 up  
 $(a, b)$   
 $(a + 2, b + 3)$

Understanding

B1

3. If the point  $(2, -8)$  is on the graph of  $y = f(x - 3) + 4$ , what point must be on the graph of  $y = f(x)$ ?

- A.  $(-1, -12)$
- B.  $(-1, -4)$
- C.  $(5, -12)$
- D.  $(5, -4)$


$(x, y)$     3 right  
                   4 up

$(x + 3, y + 4) \rightarrow (2, -8)$   
 $x = -1, y = -12$   
 $(-1, -12)$

**B2** describe how compressions and expansions of functions affect graphs and their related equations:

$$y = af(x)$$


$$y = f(kx)$$

Knowledge 

B2

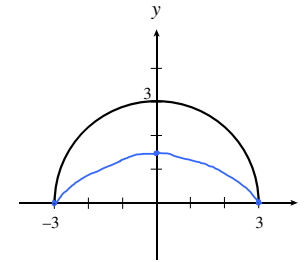
4. How is the graph of  $y = 7^{3x}$  related to the graph of  $y = 7^x$ ? x → 3x horiz comp 1/3

- A. The graph of  $y = 7^x$  has been expanded vertically by a factor of 3.
- B. The graph of  $y = 7^x$  has been compressed vertically by a factor of  $\frac{1}{3}$ .
- C. The graph of  $y = 7^x$  has been expanded horizontally by a factor of 3.
- D. The graph of  $y = 7^x$  has been compressed horizontally by a factor of  $\frac{1}{3}$ .

Understanding 

B2

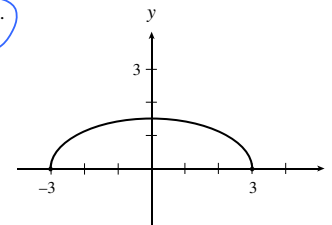
5. The graph of  $y = \sqrt{9-x^2}$  is shown below.



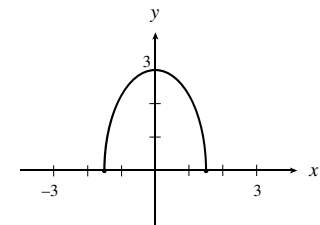
Which of the following graphs represents  $2y = \sqrt{9-x^2}$ ?

y → 2y vert comp 1/2

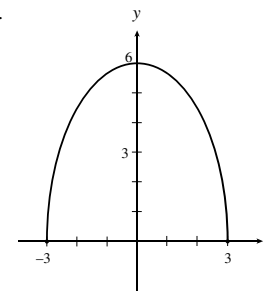
A.



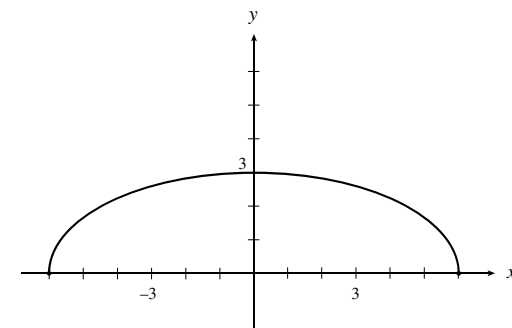
B.



C.



D.





B2, B3

6. If the graph of  $x^2 + y^2 = 4$  is vertically compressed by a factor of  $\frac{1}{5}$ , then reflected in the y-axis, determine an equation for the new graph.

A.  $x^2 + \frac{y^2}{25} = 4$

B.  $x^2 + 25y^2 = 4$

C.  $-x^2 + 25y^2 = 4$

D.  $-x^2 + \frac{y^2}{25} = 4$

$y \rightarrow 5y$

$x^2 + 25y^2 = 4$

$x \rightarrow -x$

$(-x)^2 + 25y^2 = 4$

$x^2 + 25y^2 = 4$

- B3** describe how reflections of functions in both axes and in the line  $y = x$  affect graphs and their related equations:

$y = f(-x)$

$y = -f(x)$

$y = f^{-1}(x)$

Knowledge

B3

7. The graph of  $y = -f(x)$  is a reflection of the graph of  $y = f(x)$  in

- A. the y-axis.  
 B. the x-axis.  
 C. the line  $y = x$ .  
 D. the line  $y = -x$ .

Knowledge



B3

8. What is the inverse of the relation  $y = x^3$  ?

A.  $y = \frac{1}{x^3}$

B.  $x = y^3$

C.  $y = (-x)^3$

D.  $x = y^{\frac{1}{3}}$

$$x = y^3$$

$$y = \sqrt[3]{x}$$

Understanding

B3

9. The point  $(6, -12)$  is on the graph of the function  $y = f(x)$ . Which point must be on the graph of the function  $y = 3f(-x)$ ?

- A.  $(-6, -36)$   
 B.  $(6, 36)$   
 C.  $(-6, -4)$   
 D.  $(6, 4)$

$y \rightarrow \frac{1}{3}y$  Vert exp 3  $(6, -12)$   
 $(6, -36)$   
 $x \rightarrow -x$  flip over y-axis  $(-6, -36)$

Higher Mental Processes

B3

10. If  $f(x) = \frac{2x}{x-1}$ , determine the equation of  $f^{-1}(x)$ , the inverse of  $f(x)$ .

- A.  $f^{-1}(x) = \frac{x}{x-2}$   
 B.  $f^{-1}(x) = \frac{2x}{2x-1}$   
 C.  $f^{-1}(x) = \frac{x-1}{2x}$   
 D.  $f^{-1}(x) = \frac{1}{x-2}$

$y = \frac{2x}{x-1} \Rightarrow x = \frac{2y}{y-1}$

$x(y-1) = 2y$   
 $xy - x = 2y$   
 $-x = 2y - xy$   
 $-x = y(2-x)$

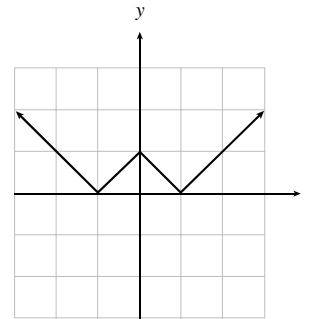
$y = \frac{-x}{2-x}$   
 $y = \frac{x}{x-2}$   $f^{-1}(x) = \frac{x}{x-2}$

Higher Mental Processes

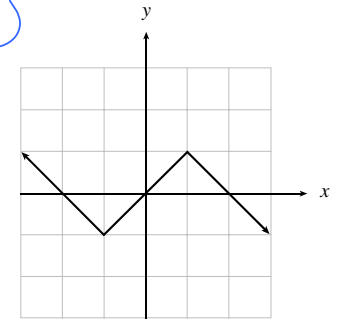
B3

11. For which graph of  $y = f(x)$  would  $f(-x) = -f(x)$ ?

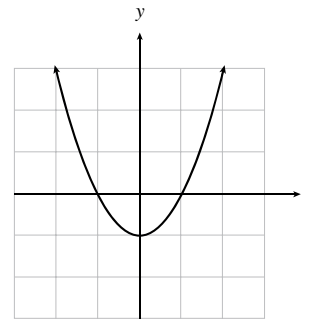
A.



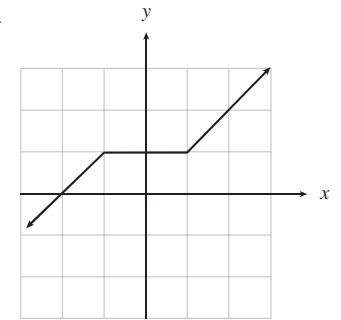
B.



C.



D.



Understanding

B3

12. When the graph of  $y = f(x)$  is transformed to the graph of  $y = f(-x)$ , on which line(s) will the invariant points lie?

- A.  $y = 0$   
 B.  $x = 0$   
 C.  $y = x$   
 D.  $y = 1, y = -1$

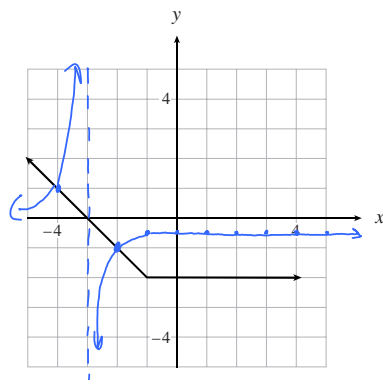
$x \rightarrow -x$  flip over y-axis  
 $x = 0$

**B4** using the graph and/or the equation of  $f(x)$ , describe and sketch  $\frac{1}{f(x)}$

Knowledge

B4

13. Given the graph of  $y = f(x)$  below, determine an equation for an asymptote for the graph of  $y = \frac{1}{f(x)}$ .

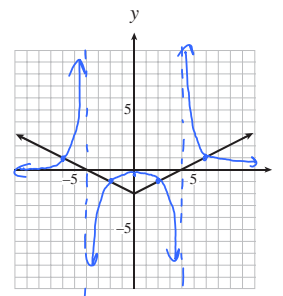


- A.  $x = 3$   
 B.  $x = -3$   
 C.  $y = -2$   
 D.  $y = 2$

Understanding

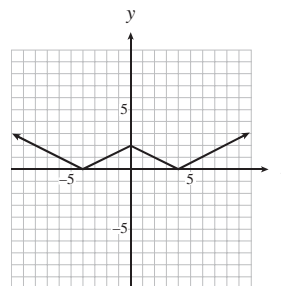
B4

14. The graph of  $y = f(x)$  is shown below.

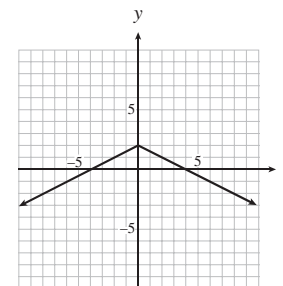


Which of the following graphs represents  $y = \frac{1}{f(x)}$ ?

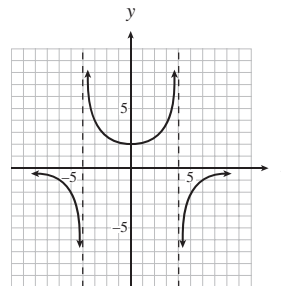
A.



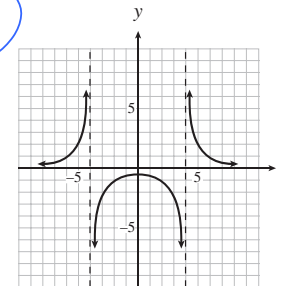
B.



C.



D.

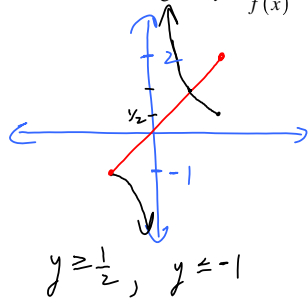


Higher Mental Processes

B4

15. If the range of  $y = f(x)$  is  $-1 \leq y \leq 2$ , what is the range of  $y = \frac{1}{f(x)}$ ?

- A.  $-1 \leq y \leq \frac{1}{2}$
- B.  $-1 \leq y \leq \frac{1}{2}, y \neq 0$
- C.  $y \geq \frac{1}{2}$  or  $y \leq -1$
- D.  $y \geq 2$  or  $y \leq -1$



Understanding

B4

16. The graph of  $y = f(x)$  is transformed to the graph of  $y = \frac{1}{f(x)}$ . If the following points are on the graph of  $y = f(x)$ , which point would be invariant?

- A. (1, 2)
- B. (2, 1)
- C. (3, 0)
- D. (0, 3)

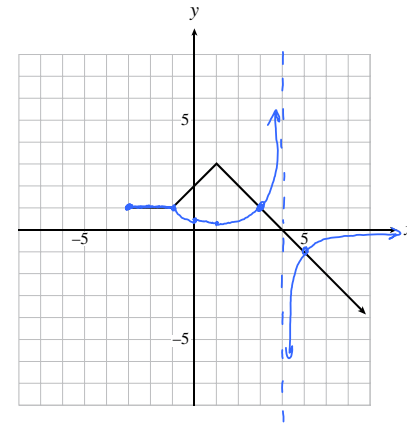
$(1, \frac{1}{2})$   
 $(2, \frac{1}{2})$   
 $(3, \infty)$   
 $(0, \frac{1}{3})$

$y \rightarrow \frac{1}{y}$

Understanding

B4

17. The graph of  $y = f(x)$  is shown below.



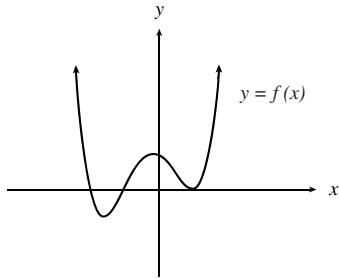
On the grid provided, sketch the graph of  $y = \frac{1}{f(x)}$ .

**B5** using the graph and/or the equation of  $f(x)$ , describe and sketch  $|f(x)|$

Understanding

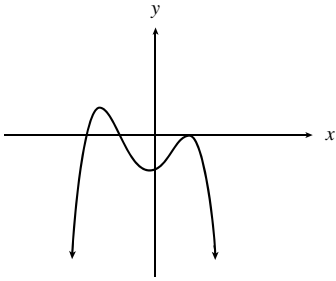
B5

18. The graph of the function  $y = f(x)$  is shown below.

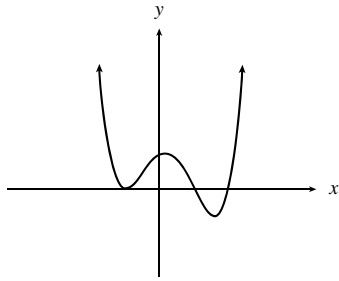


Which of the following is a graph of  $y = |f(x)|$  ?

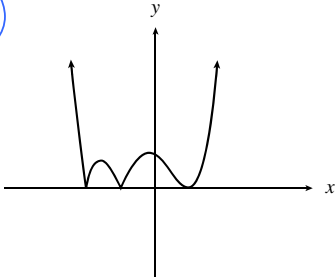
A.



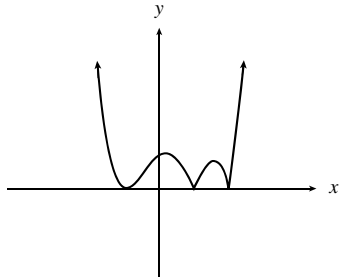
B.



C.



D.



Understanding

B5

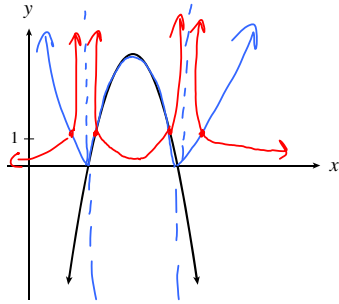
19. If the range of  $y = f(x)$  is  $-3 \leq y \leq 5$ , what is the range of  $y = |f(x)|$  ?

- A.  $-3 \leq y \leq 5$
- B.  $0 \leq y \leq 3$
- C.  $0 \leq y \leq 5$
- D.  $3 \leq y \leq 5$

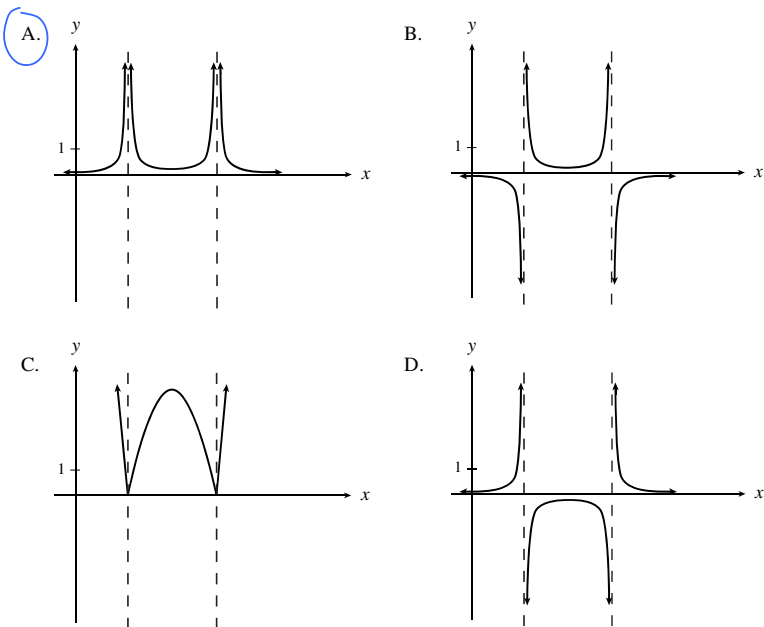
Higher Mental Processes

B5, B4

20. The graph of the function  $y = f(x)$  is shown below.



Which of the following is a graph of  $y = \frac{1}{|f(x)|}$ ?



**B6 describe and perform single transformations and combinations of transformations on functions and relations**

Clarification: The absolute value of a function and the reciprocal value of a function may also be combined with transformations.

Knowledge

B6

21. Determine an equation that will cause the graph of  $y = f(x)$  to expand vertically by a factor of 4 and shift 3 units up.

A.  $y = \frac{1}{4}f(x) + 3$

B.  $y = \frac{1}{4}f(x) - 3$

C.  $y = 4f(x) + 3$

D.  $y = 4f(x) - 3$

$y \rightarrow \frac{1}{4}y$

$y \rightarrow y - 3$

$\frac{1}{4}(y - 3) = f(x)$

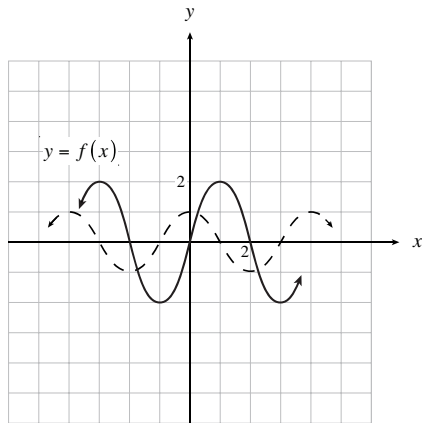
$y = 4f(x) + 3$



Understanding

B6

22. In the diagram below,  $y = f(x)$  is graphed as a broken line.



Which equation is defined by the solid line?

- A.  $y = 2f(x+1)$
- B.  $y = f(2x-1)$
- C.  $y = f(2x+1)$
- D.  $y = 2f(x-1)$

$y \rightarrow \frac{1}{2}y$

$x \rightarrow x-1$

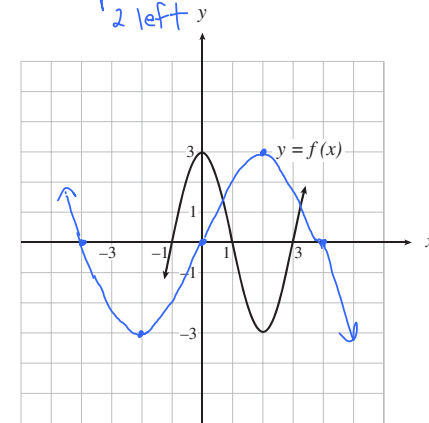
$\frac{1}{2}y = f(x-1)$   
 $y = 2f(x-1)$

Understanding

B6

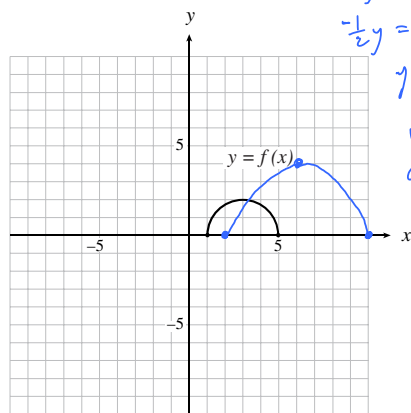
23. The graph of  $y = f(x)$  is shown below. On the grid provided, sketch the graph of  $y = -f\left(\frac{1}{2}(x+2)\right)$ .

horiz exp 2  
 flip over x  
 2 left



Higher Mental Processes  
B6

24. The graph of  $y = f(x)$  is shown below on the left. Which equation represents the graph shown on the right?



$$\frac{1}{2}y = f\left(\frac{1}{2}x\right)$$

$$-\frac{1}{2}y = f\left(\frac{1}{2}(x+6)\right)$$

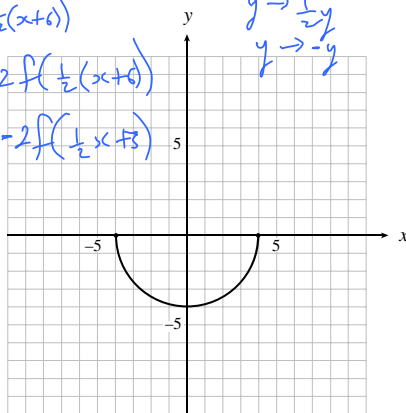
$$y = -2f\left(\frac{1}{2}(x+6)\right)$$

$$y = -2f\left(\frac{1}{2}x+3\right)$$

$$x \rightarrow \frac{1}{2}x \quad x \rightarrow x+6$$

$$y \rightarrow \frac{1}{2}y$$

$$y \rightarrow -y$$



A.  $y = -2f(2x+3)$

B.  $y = -2f(2x+6)$

C.  $y = -2f\left(\frac{1}{2}x+3\right)$

D.  $y = -2f\left(\frac{1}{2}x+6\right)$

Higher Mental Processes  
B6

25. If the point  $(6, -2)$  is on the graph  $y = f(x)$ , which point must be on the graph of  $y = \frac{1}{f(-x)+4}$ ?

A.  $\left(-10, -\frac{1}{2}\right)$

B.  $\left(-6, \frac{1}{2}\right)$

C.  $\left(-6, \frac{7}{2}\right)$

D.  $\left(-\frac{1}{6}, 2\right)$

$$y = f(x) \quad (6, -2)$$

$$x \rightarrow -x \quad (-6, -2)$$

$$y \rightarrow y-4 \quad (-6, 2)$$

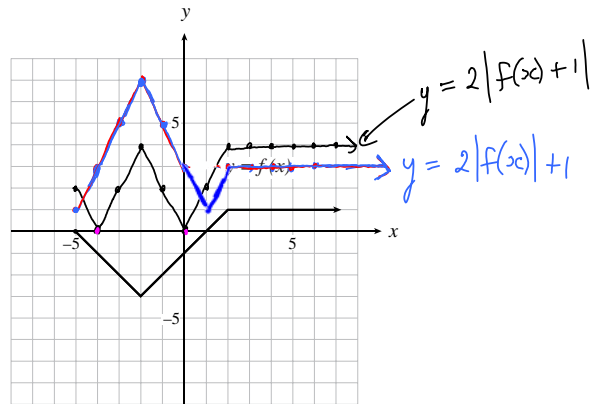
$$y \rightarrow \frac{1}{y} \quad \left(-6, \frac{1}{2}\right)$$

$$(-6, -2)$$

$$(-6, 2)$$

$$\left(-6, \frac{1}{2}\right)$$

26. The graph of  $y = f(x)$  is shown below.



*Understanding*  
B6

a) On the grid provided, sketch the graph of:

$$y = 2|f(x)| + 1$$

*Understanding*  
B6

b) On the grid provided, sketch the graph of:

$$y = 2|f(x) + 1|$$