PRINCIPLES OF MATHEMATICS 12

SAMPLE QUESTIONS

For teacher information, a graphic has been placed beside those Sample Questions in this document to indicate the type of questions which might lend themselves best to the non-calculator section (Part A, Section I) of the examination.

A PATTERNS AND RELATIONS

Patterns

— Geometric Sequences and Series

A1 ***derive and apply expressions to represent general terms for geometric growth and to solve problems***

Knowledge

A1

1. Determine the common ratio of the geometric sequence $1, -\frac{1}{3}, \frac{1}{9}, -\frac{1}{27}$.

   A. -3

   * B. $-\frac{1}{3}$

   C. $\frac{1}{3}$

   D. 3

Understanding

A1

2. Determine the 14th term of the geometric series: $6 + 12 + 24 + \ldots$

   A. 12 288
   B. 24 576
   * C. 49 152
   D. 98 304

Understanding

A1

3. The second term of a geometric series is –16 and the seventh term is 512. Determine the first term.

   A. –2
   B. 2
   C. –8
* D. 8

Higher Mental Processes

A1

4. If $x-1,\ x+6,\ 3x+4$ are the first three terms in a geometric sequence, determine the possible values of the first term.

   A. $-9,\ \frac{3}{2}$

   B. $-8,\ \frac{5}{2}$

* C. $-\frac{7}{2},\ 7$

   D. $-\frac{5}{2},\ 8$

Understanding

A1

5. In the World Dominoes tournament, 78 125 players are grouped 5 players at each table. One game is played by these 5 players and the winner at each table advances to the next round, and so on until the final game of 5 players. How many rounds would the ultimate winner have played (including the final round)?

■ SOLUTION

$$a = \frac{78\,125}{5} = 15\,625 \qquad (\text{number of tables or games in first round})$$

$$t_n = 1 \qquad r = \frac{1}{5}$$

$$n = ?$$

$$t_n = ar^{n-1}$$

$$1 = 15\,625\left(\frac{1}{5}\right)^{n-1}$$

$$\frac{1}{15\,625} = \left(\frac{1}{5}\right)^{n-1}$$

$$\left(\frac{1}{5}\right)^{6} = \left(\frac{1}{5}\right)^{n-1}$$

$$6 = n - 1$$

$$7 = n$$

$\therefore$ the winner would have played 7 rounds.

Check: Games: 15 625, 3125, 625, 125, 25, 5, 1
Players: 78 125, 15 625, 3125, 625, 125, 25, 5

■ ALTERNATE SOLUTION

$$a = 78\,125 \qquad (\text{number of players in first round})$$

$$t_n = 5 \qquad (\text{number of players in final round})$$

$$r = \frac{1}{5}$$

$$t_n = ar^{n-1}$$

$$5 = 78\,125\left(\frac{1}{5}\right)^{n-1}$$

$$n = 7$$

$\therefore$ the winner would have played 7 rounds.

6. The first and second terms of a geometric sequence have a sum of 15, while the second and third terms have a sum of 60. Use an algebraic method to find the three terms.

■ SOLUTION

$$a + ar = 15 \qquad a(1+r) = 15$$
$$ar + ar^2 = 60 \qquad ar(1+r) = 60$$

$$\therefore \frac{ar(1+r)}{a(1+r)} = \frac{60}{15}$$

$$r = 4$$

$$a + ar = 15$$

$$a + 4a = 15$$

$$5a = 15$$

$$a = 3$$

$\therefore$ sequence is: $3, 12, 48$

A2 ***derive and apply expressions to represent sums for geometric growth and to solve problems***

Knowledge
A2

7. Determine the number of terms in the series defined by $\sum_{k=12}^{38} 3(2)^{k-1}$.

   A. 26
* B. 27
   C. 37
   D. 38

Knowledge
A2

8. If the sum of n terms of the geometric sequence $4, 8, 16, \ldots$ is $S_n = \frac{4\left(1-2^5\right)}{1-2}$, determine the value of n.

   A. 2
   B. 4
* C. 5
   D. 6

Understanding
A2

9. While training for a race, a runner increases her distance by 10% each day.
   If she runs 2 km on the first day, what will be her total distance for 26 days of training? (Accurate to 2 decimal places.)

   A. 21.67
   B. 23.84
   C. 196.69
* D. 218.36

Higher Mental Processes
A2

10. If the sum of n terms in a geometric series is given by the expression $S_n = 4\left(3^n - 1\right)$, determine t_4.

A. 108
B. 160
* C. 216
D. 320

Higher Mental Processes (cross-topic)
A2

11. Evaluate: $\sum_{k=3}^{5} \log_k k^2$

A. 1
B. 2
* C. 6
D. 8

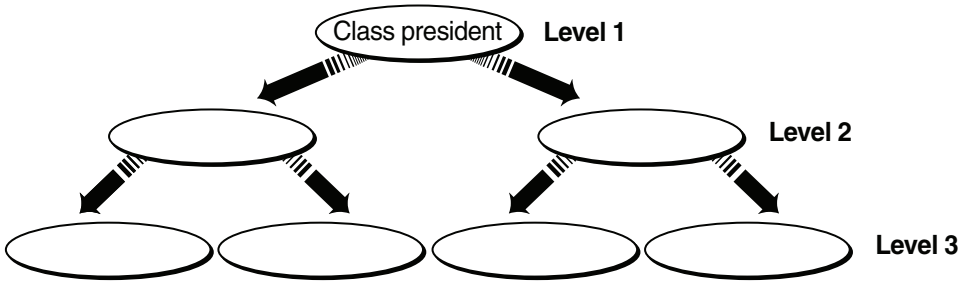
Higher Mental Processes
A2

12. The exponential function $y = 2^x$ can be used to determine the number of ancestors you have in a previous generation. For example, if $x = 2$, then $y = 4$ means that 2 generations ago you have 4 ancestors (your 4 grandparents). Determine an expression that represents the total number of ancestors you have in the last n generations.

A. $2\left(2^{n-1} - 1\right)$
B. $2^n - 1$
C. $2^{n+1} - 1$
* D. $2\left(2^n - 1\right)$

Understanding
A2

13. A graduation class informs its members of changes in plans by telephone. The president of the class calls two members, each of whom in turn calls two members, and so on, as shown in the diagram. By the 9th level, all members of the graduation class have been contacted. Determine the maximum number of students in total in the graduation class.



■ SOLUTION

Geometric series: $1+2+4+8+\ldots$

$a = 1$
$r = 2$
$n = 9$

$$S_n = \frac{a\left(1-r^n\right)}{1-r}$$

$$S_9 = \frac{1\left(1-2^9\right)}{1-2}$$

$$= 511$$

$\therefore$ there are 511 students in the graduation class.

A3 ***estimate sums of expressions represented by infinite geometric processes where the common ratio, r, is $-1 < r < 1$***

Knowledge

A3

14. Which infinite geometric series has a finite sum?

    A. $\frac{1}{2} - 1 + 2 - 4 + \ldots$

* B. $64 + 48 + 36 + 27 + \ldots$

    C. $\frac{1}{24} + \frac{1}{12} + \frac{1}{6} + \frac{1}{3} + \ldots$

    D. $16 - 20 + 25 - 31.25 + \ldots$

Understanding

A3

15. Determine the sum of the infinite geometric series: $800 + 300 + \frac{225}{2} + \ldots$

* A. 1280
   B. 1212.5
   C. 1254.69
   D. no finite sum

Higher Mental Processes
A3

16. Evaluate: $\sum_{n=1}^{\infty} \frac{1}{3^n}$

A. $\frac{1}{3}$

* B. $\frac{1}{2}$

C. $\frac{2}{3}$

D. 1

Higher Mental Processes
A3

17. For what values of x, $x \neq -1$, will the following infinite geometric series have a finite sum?

$$(x+1)+(x+1)^2+(x+1)^3+\ldots$$

A. $-1 < x < 0$
B. $-1 < x < 1$
C. $-3 < x < 1$
* D. $-2 < x < 0$

18. A ball is dropped from a height of 5 m. After each bounce, it rises to 60% of its previous height.

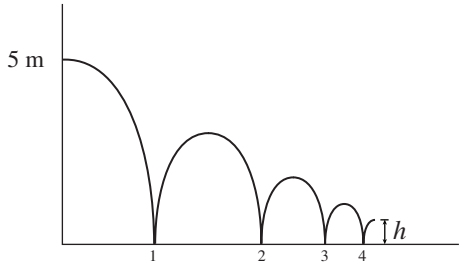
A1 a) What is the maximum height the ball will reach after it hits the ground for the 4th time?

A2 b) What is the total vertical distance the ball travels by the time the ball hits the ground for the 7th time?

A3 c) What is the total vertical distance the ball travels before it comes to rest?

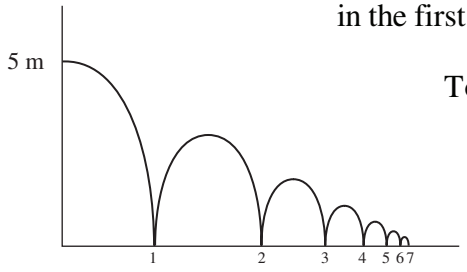
■ SOLUTION

a)



$$h = t_5 = 5(0.6)^4 = 0.648$$

b) ***Note:*** For S_n, consider t as the total vertical distance in the first complete interval, i.e. $3 + 3 = 6$.



$$\text{Total vertical distance} = S_n + 5 \text{ m}$$

$$= \frac{6\left(1 - 0.6^6\right)}{1 - 0.6} + 5 \text{ m}$$

$$= 14.300 + 5 \text{ m}$$

$$= 19.300 \text{ m}$$

c)

$$\text{Total vertical distance} = S + 5 \text{ m}$$

$$= \frac{6}{1 - 0.6} + 5 \text{ m}$$

$$= 20 \text{ m}$$

Note: *There are a variety of solution methods for these problems.*

A PATTERNS AND RELATIONS

Variables and Equations
Relations and Functions
— *Logarithms and Exponents*

A4 ***solve exponential equations having bases that are powers of one another***

Understanding 

A4

1. Solve for x: $81^{x-1} = \left(\frac{1}{27}\right)^{x-4}$

   A. -8

   B. -3

   C. $-\frac{3}{7}$

   * D. $\frac{16}{7}$

Higher Mental Processes 

A4

2. Solve: $9^{x+2} = \left(3^{4x-3}\right)\left(3^{5}\right)$

   A. 0

   * B. 1

   C. $\frac{17}{19}$

   D. $\frac{19}{18}$

A5 ***solve and verify exponential and logarithmic equations***

Knowledge 

A5

3. Solve for x: $5 = 3^x$

   A. $x = \log_5 3$

   * B. $x = \log_3 5$

   C. $x = 3^5$

   D. $x = 5^3$

Higher Mental Processes 

A5

4. Solve for x: $ab^x = c$

   A. $x = \dfrac{\log c}{\log a + \log b}$

   B. $x = \dfrac{\log c + \log a}{\log b}$

   * C. $x = \dfrac{\log c - \log a}{\log b}$

   D. $x = \dfrac{\log c}{\log b} - \log a$

Higher Mental Processes
A5

5. Solve algebraically using logarithms: $2^x = 3\left(5^{x+1}\right)$
(Answer accurate to at least 2 decimal places.)

■ SOLUTION

$$2^x = 3\left(5^{x+1}\right)$$

$$\log 2^x = \log\left(3\left(5^{x+1}\right)\right)$$

$$x\log 2 = \log 3 + (x+1)\log 5$$

$$x\log 2 = \log 3 + x\log 5 + \log 5$$

$$x\log 2 - x\log 5 = \log 3 + \log 5$$

$$x(\log 2 - \log 5) = \log 3 + \log 5$$

$$x = \frac{\log 5}{\log 2 - \log 5}$$

$$x = -2.96$$

Understanding 
A5

6. Solve for x: $\log(3-x) + \log(3+x) = \log 5$

A. $x = -2$
B. $x = 2$
* C. $x = \pm 2$
D. no solution

Understanding

A5

7. Solve: $\log_2 8 + \log_3 \frac{1}{3} = \log_4 x$

   A. $\frac{1}{64}$

   B. $\frac{1}{16}$

* C. 16

   D. 64

Higher Mental Processes

A5

8. Solve the following: $\log_2 \left(\log_4 \left(\log_5 x\right)\right) = -1$

   A. $\frac{1}{25}$

   B. 5

* C. 25

   D. 125

Understanding
A5

9. Solve algebraically: $2\log_4 x - \log_4 (x+3) = 1$

■ SOLUTION

$$\log_4 x^2 - \log_4 (x+3) = 1$$

$$\log_4 \frac{x^2}{x+3} = 1$$

$$4 = \frac{x^2}{x+3}$$

$$4x + 12 = x^2$$

$$0 = x^2 - 4x - 12$$

$$0 = (x-6)(x+2)$$

$$x = 6, \qquad x = -2$$

$$\Downarrow$$

reject

$$\therefore \; x = 6$$

A6 ***solve and verify exponential and logarithmic identities***

Understanding 

A6

10. Simplify: $\log_2 4^x$

A. x

* B. $2x$

C. 2^x

D. x^2

Understanding 

A6

11. Write as a single logarithm: $3 + \frac{1}{2}\log_2 x - 3\log_2 y$

A. $\log_2\left(\frac{1000\sqrt{x}}{y^3}\right)$

* B. $\log_2\left(\frac{8\sqrt{x}}{y^3}\right)$

C. $\log_2\left(1000 + \sqrt{x} - y^3\right)$

D. $\log_2\left(8 + \sqrt{x} - y^3\right)$

Understanding 

A6

12. If $\log_4 x = a$, determine $\log_{16} x$ in terms of a.

A. $\frac{a}{4}$

* B. $\frac{a}{2}$

C. $2a$

D. $4a$

Higher Mental Processes 

A6

13. If $\log 2 = a$, $\log 3 = b$, determine an expression for $\log 2400$.

A. $2a^3b$

* B. $3a + b + 2$

C. $3a + b + 100$

D. $a^3 + b + 2$

Higher Mental Processes 

A6

14. Simplify: $a^{\log_a 8 + \log_a 2}$

A. 10

* B. 16

C. a^{10}

D. a^{16}

Higher Mental Processes 

A6

15. Determine the value of $\log_n ab^2$ if $\log_n a = 5$ and $\log_n b = 3$.

* A. 11
B. 14
C. 16
D. 45

Higher Mental Processes 

A5, A6

16. Given $\log_a 2 = x$ and $\left(\log_a 8\right)\left(a^{\log_a x}\right) = 12$, solve for a.

A. 2
B. ± 2
* C. $\sqrt{2}$
D. $\pm\sqrt{2}$

A13 ***change functions from exponential form to logarithmic form and vice versa***

Knowledge 

A13

17. Change to exponential form: $\log_k \ell = m$

A. $\ell = m^k$

* B. $\ell = k^m$

C. $k = m^\ell$

D. $k = \ell^m$

Knowledge 

A13

18. If (a, b) is on the graph of $y = 3^x$, which point must be on the graph of $y = \log_3 x$?

A. (a, b)

* B. (b, a)

C. $(3a, b)$

D. $(a, 3b)$

Understanding (cross topic) 

A13

19. Determine the inverse of $f(x) = 3^{x-1} - 2$.

* A. $f^{-1}(x) = \log_3(x+2)+1$

B. $f^{-1}(x) = \log_3(x+2)-1$

C. $f^{-1}(x) = \log_3(x-1)+2$

D. $f^{-1}(x) = \log_3(x-1)-2$

A14 ***model, graph, and apply exponential functions to solve problems***

Clarification: Students should be familiar with using base e in continuous growth and decay problems.

Understanding 

A14

20. If $5000 is invested at 7.2% per annum compounded monthly, which equation can be used to determine the number of years, t, for the investment to increase to $8000?

A. $8000 = 5000(1.072)^{t}$

B. $8000 = 5000(1.006)^{t}$

C. $8000 = 5000(1.072)^{12t}$

* D. $8000 = 5000(1.006)^{12t}$

Understanding

A14

21. The population of a particular country is 25 million. Assuming the population is growing continuously, the population, P, in millions, t years from now can be determined by the formula $P = 25e^{0.022t}$. What will be the population, in millions, 20 years from now?

A. 29.90
B. 37.97
C. 38.63
* D. 38.82

Understanding
A14

22. The population of a nest of ants can multiply threefold (triple) in 8 weeks. If the population is now 12 000, how many weeks will it take for the population to reach 300 000 ants?

    (Solve algebraically using logarithms. Answer accurate to at least 2 decimal places.)

■ SOLUTION

$$300\,000 = 12\,000\left(3\right)^{\frac{t}{8}}$$

$$25 = 3^{\frac{t}{8}}$$

$$\log 25 = \log 3^{\frac{t}{8}}$$

$$\log 25 = \frac{t}{8}\log 3$$

$$t = \frac{8\log 25}{\log 3}$$

$$t = 23.44 \text{ weeks}$$

Understanding
A14

23. The radioactivity of a certain substance decays by 20% in 30 hours. What is the half-life of the substance?

■ SOLUTION

If 20% of the radioactivity decays, then 80% remains.

$$80 = 100\left(\frac{1}{2}\right)^{\frac{30}{n}}$$

$$0.80 = \left(\frac{1}{2}\right)^{\frac{30}{n}}$$

$$\log 0.80 = \frac{30}{n}\log\frac{1}{2}$$

$$n = \frac{30\log\frac{1}{2}}{\log 0.80}$$

$$n \approx 93.19 \text{ hours}$$

Understanding

A14

24. The intensity of light reduces by 7% for every 3 metres below the surface of the water. At what depth will the light intensity be reduced to 60% of its original amount?

■ SOLUTION

$$60 = 100(1-0.07)^{\frac{x}{3}}$$

$$0.6 = 0.93^{\frac{x}{3}}$$

$$\log 0.6 = \log 0.93^{\frac{x}{3}}$$

$$\log 0.6 = \frac{x}{3}\log 0.93$$

$$3\frac{\log 0.6}{\log 0.93} = x$$

$$x = 21.11702137$$

$$x \approx 21.12 \text{ m}$$

■ ALTERNATE STEPS

$$0.6 = 0.93^{\frac{x}{3}}$$

$$\log_{0.93} 0.6 = \frac{x}{3}$$

$$x \approx 21.12 \text{ m}$$

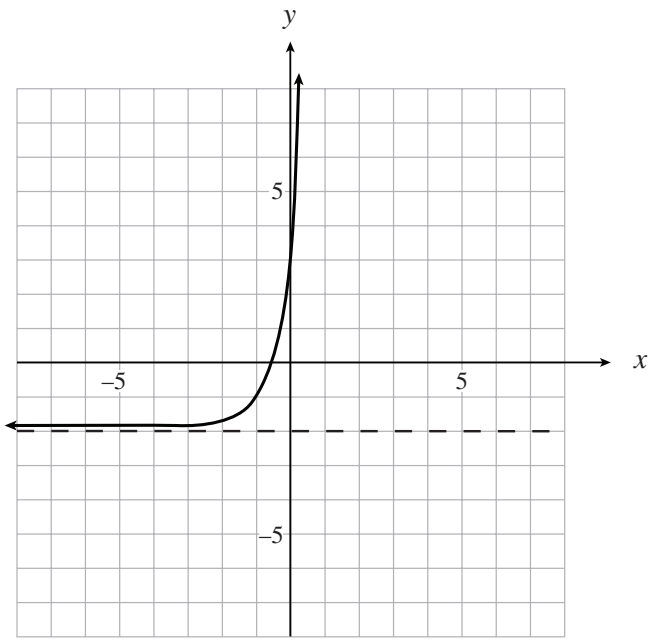
25. Graph $\log_5(y+2) = x+1$ on the grid below. State any asymptotes and determine the x- and y-intercepts.

■ SOLUTION

Since $\log_5(y+2) = x+1$

$$y+2 = 5^{x+1}$$

$$y = 5^{x+1} - 2$$



asymptote: $y = -2$

x-intercept: set $y = 0$ **or** x-intercept: set $y = 0$ in original equation

$$0 = 5^{x+1} - 2$$

$$2 = 5^{x+1}$$

$$\log 2 = (x+1)\log 5$$

$$\frac{\log 2}{\log 5} = x+1$$

$\therefore$ x-intercept is $\dfrac{\log 2}{\log 5} - 1$

or $\log_5 2 - 1 = -0.57$

$$\log_5(0+2) = x+1$$

$$\log_5 2 = x+1$$

$$\therefore x = \log_5 2 - 1$$

$$= -0.57$$

y-intercept: set $x = 0$

$$y = 5^{0+1} - 2$$

$\therefore$ y-intercept is 3

26. The population of Canada is 30 million people and is growing at an annual rate of 1.4%. The population of Germany is 80 million people and is decreasing at an annual rate of 1.7%. In how many years will the population of Canada be equal to the population of Germany?

    (Solve algebraically using logarithms. Answer accurate to at least 2 decimal places.)

■ SOLUTION

$$30(1+0.014)^n = 80(1-0.017)^n$$

$$30(1.014)^n = 80(0.983)^n$$

$$\log 30 + n\log(1.014) = \log 80 + n\log(0.983)$$

$$n\left(\log(1.014) - \log(0.983)\right) = \log 80 - \log 30$$

$$n = \frac{\log 80 - \log 30}{\log(1.014) - \log(0.983)}$$

$$n = 31.59 \text{ years}$$

A15 ***model, graph, and apply logarithmic functions to solve problems***

Understanding 

A15

27. Determine the domain of the function $y = \log(2x + 3)$.

* A. $x > -\frac{3}{2}$

B. $x > -\frac{2}{3}$

C. $x > \frac{2}{3}$

D. $x > \frac{3}{2}$

Understanding

A15

28. In 1976, an earthquake in Guatemala had a magnitude of 7.5 on the Richter scale and in 1960, an earthquake in Morocco had a magnitude of 5.8. How many times as intense was the 1976 Guatemalan earthquake compared to the 1960 Moroccan earthquake?

A. 1.29

B. 1.7

C. $10^{1.29}$

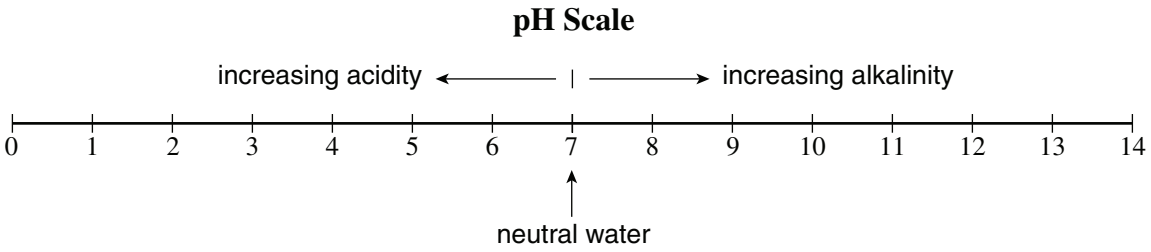
* D. $10^{1.7}$

Understanding

A15

29. In chemistry, the pH scale measures the acidity (0–7) or alkalinity (7–14) of a solution. It is a logarithmic scale in base 10. Thus, a pH of 5 is 10 times more acidic than a pH of 6.

    Solution A has a pH of 5.7. Solution B is 1260 times more acidic than Solution A. Find the pH of solution B.

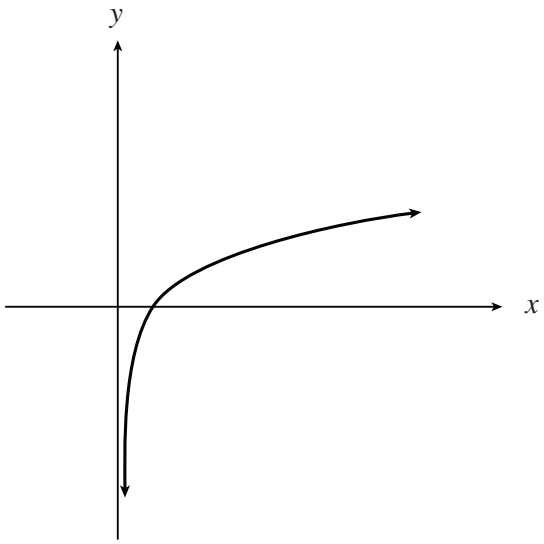


* A. 2.6
B. 4.4
C. 7.0
D. 8.8

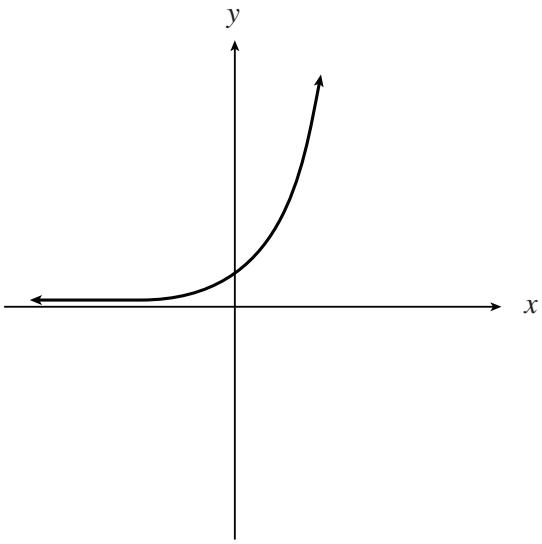
A15

30. If $0 < a < 1$, which of the following is the best graph of $y = \log_a x$?

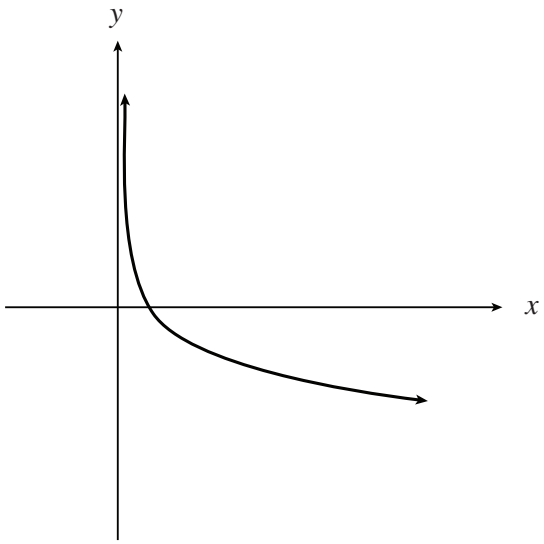
A.



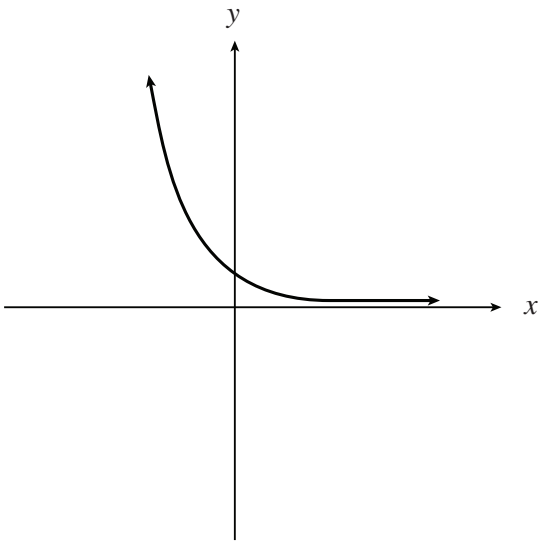
B.



* C.



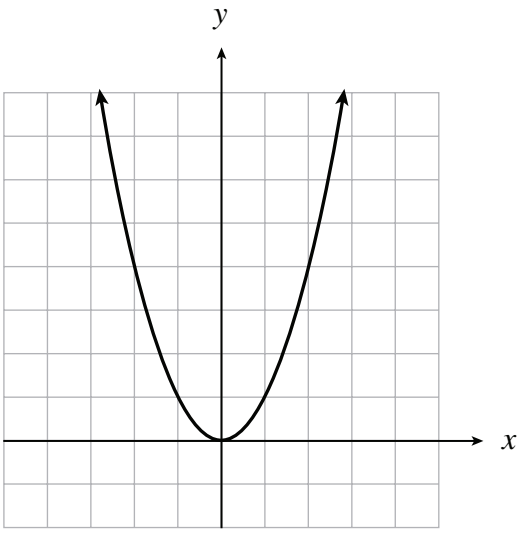
D.



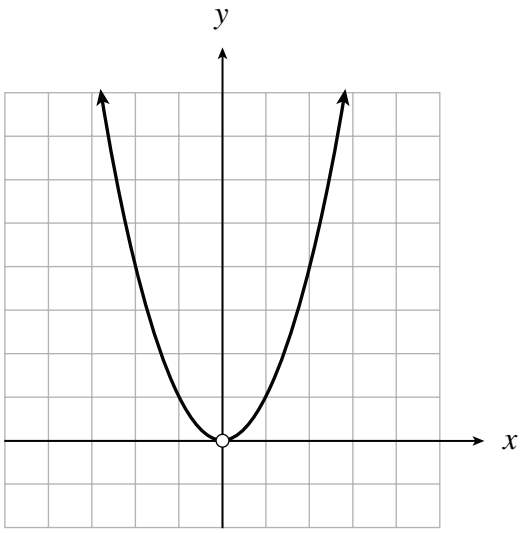
A15

31. Which of the following is a graph of $\log_x y = 2$?

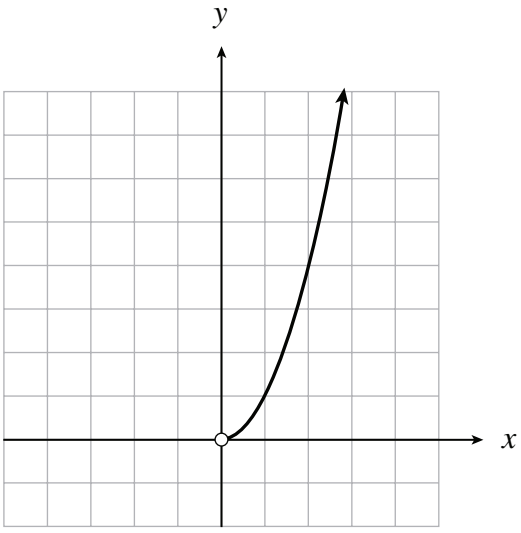
A.



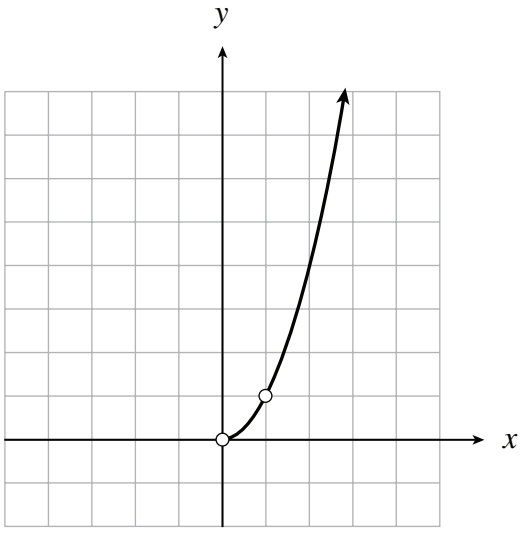
B.



C.



* D.



A PATTERNS AND RELATIONS

Variables and Equations
Relations and Functions
— *Trigonometry*

A7 ***distinguish between degree and radian measure, and solve problems using both***

Understanding 

A7

1. Convert $\frac{5\pi}{2}$ radians to degrees.

   A. 90°
   B. 180°
   C. 270°
* D. 450°

Understanding 

A7

2. A circle has a radius of 20 cm. Determine the length of the arc subtended by a central angle of 135°.

   A. $\frac{3\pi}{4}$ cm

   B. 5π cm

* C. 15π cm

   D. $\frac{80}{3\pi}$ cm

A7

3. The terminal arm of angle θ in standard position passes through the point $\left(-\sqrt{3}, -1\right)$. Determine the length of arc AB, as shown below.

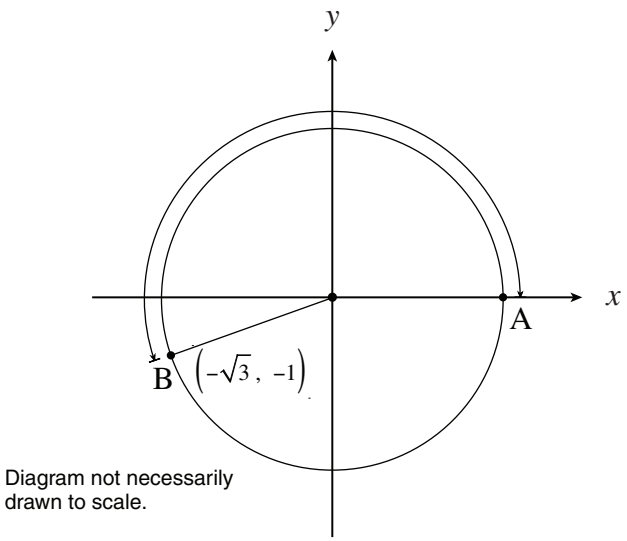


Diagram not necessarily drawn to scale.

A. $\frac{5\pi}{6}$

B. $\frac{7\pi}{6}$

* C. $\frac{7\pi}{3}$

D. $\frac{8\pi}{3}$

A8 ***determine the exact and the approximate values of trigonometric ratios for any multiples of 0°, 30°, 45°, 60° and 90°, and 0 rad,*** $\frac{\pi}{6}$ ***rad,*** $\frac{\pi}{4}$ ***rad,*** $\frac{\pi}{3}$ ***rad,*** $\frac{\pi}{2}$ ***rad***

Clarification: This includes negative angles and angles greater than 2π *rad or 360 degrees.*

Understanding 

A8

4. Evaluate: $\sec\frac{4\pi}{3}$

* A. -2

B. $-\frac{2}{\sqrt{3}}$

C. $\frac{2}{\sqrt{3}}$

D. 2

Understanding 

A8

5. Determine the exact value of $\tan\frac{8\pi}{3}$.

* A. $-\sqrt{3}$

B. $-\frac{1}{\sqrt{3}}$

C. $\frac{1}{\sqrt{3}}$

D. $\sqrt{3}$

Understanding 

A8

6. Determine the exact value of $\sin\left(-\frac{3\pi}{4}\right)$.

A. $-\sqrt{2}$

* B. $-\frac{1}{\sqrt{2}}$

C. $\frac{1}{\sqrt{2}}$

D. $\sqrt{2}$

A9 ***solve first and second degree trigonometric equations over a specified domain***
- ***algebraically***
- ***graphically***

Clarification: It should be noted that equations could be solved over any specified domain, ***not*** *restricted to* $0 \le x < 2\pi$. *This includes solutions to trigonometric equations involving multiple angles,* $k\theta$, *where* $k = \frac{1}{3}, \frac{1}{2}, 2, 3, 4, 5$. *Some solutions to trigonometric equations may involve the use of identities.*

Understanding 

A9

7. Solve: $2\cos^2 x - \cos x - 1 = 0,\ 0 \le x < 2\pi$

A. $x = 0, \frac{5\pi}{6}, \frac{7\pi}{6}$

* B. $x = 0, \frac{2\pi}{3}, \frac{4\pi}{3}$

C. $x = \frac{x}{6}, \pi, \frac{11\pi}{6}$

D. $x = \frac{\pi}{3}, \pi, \frac{5\pi}{3}$

Understanding 

A9

8. Solve: $\csc x = 2,\ 0 \le x < 2\pi$

* A. $x = \frac{\pi}{6}, \frac{5\pi}{6}$

B. $x = \frac{\pi}{6}, \frac{11\pi}{6}$

C. $x = \frac{\pi}{3}, \frac{2\pi}{3}$

D. $x = \frac{\pi}{3}, \frac{4\pi}{3}$

Understanding

A9

9. Solve: $2\sin x = \cos 3x$, where $0 \le x < 2\pi$

* A. 0.31, 3.45
B. 2.83, 5.98
C. 0.39, 2.75, 4.03, 5.30
D. 0.98, 2.16, 3.55, 5.89

Higher Mental Processes 

A9

10. Determine the number of solutions in the interval $0 \le x < 2\pi$ for:

$$\sin ax = \frac{1}{3}, \quad a \text{ is an integer, where } a \ge 1$$

A. 2

B. $\frac{a}{2}$

C. a

* D. $2a$

Higher Mental Processes

A9

11. Solve: $\sin 2x = \frac{1}{\sqrt{2}}$, where $0 \le x < 2\pi$

A. $x = \frac{\pi}{8}, \frac{3\pi}{8}$

B. $x = \frac{\pi}{4}, \frac{3\pi}{4}$

* C. $x = \frac{\pi}{8}, \frac{3\pi}{8}, \frac{9\pi}{8}, \frac{11\pi}{8}$

D. $x = \frac{\pi}{4}, \frac{3\pi}{4}, \frac{5\pi}{4}, \frac{7\pi}{4}$

Understanding

A9

12. Solve algebraically, giving exact values, where $0 \le x < 2\pi$.

$$\sin x = \cos 2x$$

■ SOLUTION

$$\sin x = \cos 2x$$

$$\sin x = 1 - 2\sin^2 x$$

$$2\sin^2 x + \sin x - 1 = 0$$

$$(2\sin x - 1)(\sin x + 2) = 0$$

$$\sin x = \frac{1}{2}, \quad \sin x = -1$$

$$x = \frac{\pi}{6}, \frac{5\pi}{6}, \frac{3\pi}{2}$$

Understanding
A9

13. Solve algebraically, giving exact values, where $-\frac{\pi}{2} < x < \frac{\pi}{2}$:

$$2\tan x \cos x - \sqrt{3}\tan x = 0$$

■ SOLUTION

$$2\tan x \cos x - \sqrt{3}\tan x = 0$$

$$\tan x\left(2\cos x - \sqrt{3}\right) = 0$$

$\therefore \tan x = 0$ or $2\cos x - \sqrt{3} = 0$

$\therefore\ x = 0$ $\qquad\qquad \cos x = \frac{\sqrt{3}}{2}$

$$x = -\frac{\pi}{6},\quad x = \frac{\pi}{6}$$

$\therefore\ x = 0$ or $x = -\frac{\pi}{6}$ or $x = \frac{\pi}{6}$

14. Solve algebraically, giving exact values: $\sin\frac{1}{3}x = \frac{\sqrt{3}}{2}$

a) where $0 \le x < 2\pi$

■ SOLUTION

$$\sin\frac{1}{3}x = \frac{\sqrt{3}}{2}$$

$\therefore \ \frac{1}{3}x = \frac{\pi}{3}$ or $\frac{1}{3}x = \frac{2}{3}\pi$

$x = \frac{3\pi}{3}$ $\qquad$ $x = \frac{6\pi}{3}$

$x = \pi$ $\qquad$ $x = 2\pi$

$\downarrow$

reject

not in domain

$\therefore \ x = \pi$

b) over the set of real numbers

■ SOLUTION

$x = \pi + 6\pi n$, $x = 2\pi + 6\pi n$, where n is an integer

A10 ***determine the general solutions to trigonometric equations where the domain is the set of real numbers***

Clarification: It is expected that students will indicate that "n is an integer" when giving general solutions. This includes solutions to trigonometric equations involving multiple angles, $k\theta$, where $k = \frac{1}{3}, \frac{1}{2}, 2, 3, 4, 5$. Also, when asked to solve a trigonometric equation "over the set of real numbers", it is expected that students must use radian measure. Some solutions to trigonometric equations may involve the use of identities.

Understanding
A10

15. Determine the general solution: $\sin 2x = -\frac{1}{2}$

A. $\frac{7\pi}{12} + 2n\pi, \frac{11\pi}{12} + 2n\pi$, n is an integer

* B. $\frac{7\pi}{12} + n\pi, \frac{11\pi}{12} + n\pi$, n is an integer

C. $\frac{13\pi}{12} + 2n\pi, \frac{21\pi}{12} + 2n\pi$, n is an integer

D. $\frac{13\pi}{12} + n\pi, \frac{21\pi}{12} + n\pi$, n is an integer

Understanding
A10

16. Solve $\cos^2 x = \cos x$ over the set of real numbers. (Give exact value solutions.)

■ SOLUTION

$$\cos^2 x - \cos x = 0$$
$$\cos x(\cos x - 1) = 0$$

$\cos x = 0$ $\qquad$ $\cos x - 1 = 0$

$\cos x = 1$

$x = \frac{\pi}{2}, \frac{3\pi}{2}$ $\qquad$ $x = 0$

$x = \frac{\pi}{2} + 2n\pi$ $\qquad$ $x = 0 + 2n\pi$

$x = \frac{3\pi}{2} + 2n\pi$ $\qquad$ $x = 2n\pi$

or

$x = \frac{\pi}{2} + n\pi$ $\qquad$ $x = 2n\pi$, where n is an integer

Understanding
A10

17. The two smallest positive solutions of $\sin 3x = 0.7$ are $x = 0.26$ and $x = 0.79$. Determine the general solution for $\sin 3x = 0.7$.

■ SOLUTION

$x = 0.26 + \frac{2n\pi}{3}, \ 0.79 + \frac{2n\pi}{3}$, where n is an integer

Understanding
A10

18. Solve algebraically $6\sin^2 x - \sin x - 2 = 0$ over the set of real numbers.

(Give exact value solutions where possible, otherwise answer accurate to two decimal places.)

■ SOLUTION

$$6\sin^2 x - \sin x - 2 = 0$$

$$(3\sin x - 2)(2\sin x + 1) = 0$$

$$\sin x = \frac{2}{3} \quad \text{or} \quad \sin x = -\frac{1}{2}$$

$x = 0.73$ or $x = 2.41$ $\qquad$ $x = \frac{7\pi}{6}$ or $x = \frac{11\pi}{6}$

$x = 0.73 + 2\pi n$ $\qquad$ $x = \frac{7\pi}{6} + 2\pi n$

$x = 2.41 + 2\pi n$ $\qquad$ $x = \frac{11\pi}{6} + 2\pi n$, where n is an integer

19. Solve algebraically $\sin 2x - 2\cos^2 x = 0$ over the set of real numbers.
(Give exact value solutions.)

■ SOLUTION

$$\sin 2x - 2\cos^2 x = 0$$
$$2\sin x \cos x - 2\cos^2 x = 0$$
$$2\cos x(\sin x - \cos x) = 0$$

$\cos x = 0$ $\qquad$ $\sin x = \cos x$

$x = \frac{\pi}{2}, \frac{3\pi}{2}$ $\qquad$ $\frac{\sin x}{\cos x} = 1$

$\tan x = 1$

$x = \frac{\pi}{4}, \frac{5\pi}{4}$

$x = \frac{\pi}{2} + 2\pi n$ $\qquad$ $x = \frac{\pi}{4} + \pi n$

$x = \frac{3\pi}{2} + 2\pi n$

or

$x = \frac{\pi}{2} + \pi n$ $\qquad$ $x = \frac{\pi}{4} + \pi n$, where n is an integer

A11 ***analyze trigonometric identities***

- ***graphically***
- ***algebraically for general cases***

Clarification: It should be noted that a numerical or graphical justification of an identity does not prove the identity.

Understanding 

A11

20. Determine the restriction(s) for the expression $\frac{\tan\theta}{2\cos\theta - 1}$.

A. $\cos\theta \neq \frac{1}{2}$

B. $\sin\theta \neq 0$

C. $\sin\theta \neq 0,\ \cos\theta \neq \frac{1}{2}$

* D. $\cos\theta \neq 0,\ \cos\theta \neq \frac{1}{2}$

Understanding 

A11

21. Determine an expression equivalent to $\tan^2\theta\csc\theta + \frac{1}{\sin\theta}$.

A. $\sec^3\theta$

B. $\csc^3\theta$

C. $\csc^2\theta\sec\theta$

* D. $\sec^2\theta\csc\theta$

Understanding

A11

22. Determine an expression equivalent to $\dfrac{\tan\theta \csc^2\theta}{\sec^2\theta}$.

A. $\tan\theta$

* B. $\cot\theta$

C. $\tan^2\theta$

D. $\tan^3\theta$

Higher Mental Processes

A11

23. Prove the identity:

$$\frac{\cos x + \cot x}{\sec x + \tan x} = \cos x \cot x$$

■ SOLUTION

LEFT SIDE	RIGHT SIDE
$\dfrac{\cos x + \cot x}{\sec x + \tan x}$	$\cos x \cot x$
$\dfrac{\sin x \cos x}{\sin x \cos x} \dfrac{\left(\cos x + \dfrac{\cos x}{\sin x}\right)}{\left(\dfrac{1}{\cos x} + \dfrac{\sin x}{\cos x}\right)}$	$\cos x \dfrac{\cos x}{\sin x}$
	$\dfrac{\cos^2 x}{\sin x}$
$\dfrac{\sin x \cos^2 x + \cos^2 x}{\sin x + \sin^2 x}$	
$\dfrac{\cos^2 x(\sin x + 1)}{\sin x(1 + \sin x)}$	
$\dfrac{\cos^2 x}{\sin x}$	

LS = RS

24. Prove the identity:

$$\frac{2\cos x + 2\cos^2 x}{\sin 2x} = \frac{\sin x}{1-\cos x}$$

■ SOLUTION

LEFT SIDE	RIGHT SIDE
$\frac{2\cos x + 2\cos^2 x}{\sin 2x}$	$\frac{\sin x}{1-\cos x}$
$\frac{2\cos x(1+\cos x)}{2\sin x\cos x}$	
$\left(\frac{1+\cos x}{\sin x}\right)\left(\frac{1-\cos x}{1-\cos x}\right)$	
$\frac{(1+\cos x)(1-\cos x)}{\sin x(1-\cos x)}$	
$\frac{1-\cos^2 x}{\sin x(1-\cos x)}$	
$\frac{\sin^2 x}{\sin x(1-\cos x)}$	
$\frac{\sin x}{1-\cos x}$	

LS = RS

A12 ***use sum, difference, and double angle identities for sine and cosine to verify and simplify trigonometric expressions***

Clarification: It should be noted that a numerical or graphical justification of an identity does not prove the identity. It should also be noted that students should be able to combine reciprocal and rational identities with double angle identities; for example:

$$\tan 2\theta = \frac{\sin 2\theta}{\cos 2\theta},\ \csc 2\theta = \frac{1}{\sin 2\theta},\ \sec 2\theta = \frac{1}{\cos 2\theta}$$

Understanding 

A12

25. Determine an expression equivalent to $\cos(\pi + 2A)$.

* A. $-\cos 2A$
B. $\cos 2A$
C. $-\sin 2A$
D. $\sin 2A$

Understanding 

A12

26. Simplify: $\cos 2x \cos x + \sin 2x \sin x$

* A. $\cos x$
B. $\sin x$
C. $\cos 3x$
D. $\sin 3x$

Understanding 

A12

27. Simplify: $\dfrac{2\sin\theta}{\sin 2\theta}$

A. 1
B. $\cos\theta$
C. $\csc\theta$
* D. $\sec\theta$

28. Prove the identity:

$$\frac{\tan x + \sin x}{1 + \cos x} = \frac{1}{\csc 2x} - \frac{\tan x}{\sec 2x}$$

■ SOLUTION

LEFT SIDE	RIGHT SIDE
$\frac{\tan x + \sin x}{1 + \cos x}$	$\frac{1}{\csc 2x} - \frac{\tan x}{\sec 2x}$
$\frac{\left(\frac{\sin x}{\cos x} + \sin x\right)\cos x}{(1 + \cos x)\cos x}$	$\sin 2x - \tan x \cos 2x$
$\frac{\sin x + \sin x \cos x}{(1 + \cos x)\cos x}$	$2 \sin x \cos x - \frac{\sin x}{\cos x}\left(2\cos^2 x - 1\right)$
$\frac{\sin x (1 + \cos x)}{(1 + \cos x)\cos x}$	$2 \sin x \cos x - 2 \sin x \cos x + \frac{\sin x}{\cos x}$
$\frac{\sin x}{\cos x}$	$\frac{\sin x}{\cos x}$

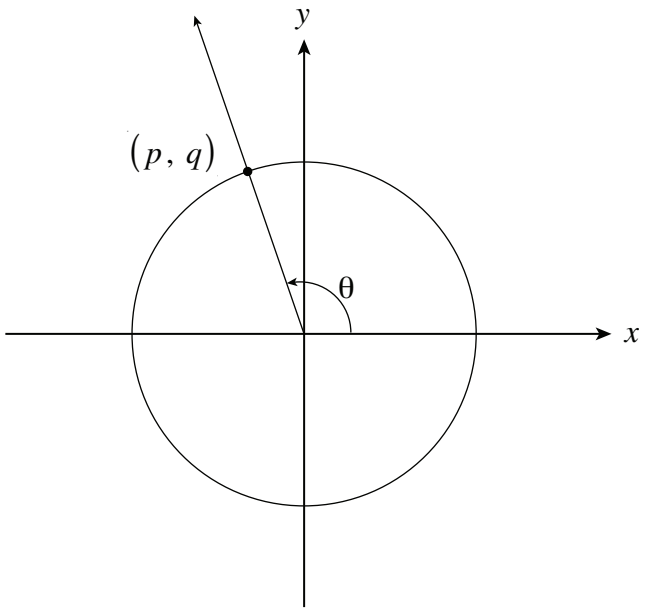
LS = RS

A16 ***describe the three primary trigonometric functions as circular functions with reference to the unit circle and an angle in standard position***

Knowledge

A16

29. The point (p, q) is the point of intersection of the terminal arm of angle θ in standard position and the unit circle as shown in the diagram. Which expression represents $\tan\theta$?



A. p

B. q

C. $\frac{p}{q}$

* D. $\frac{q}{p}$

Understanding

A16

30. The terminal arm of angle θ in standard position passes through the point $(-2,\ 5)$. Determine the value of $\sec\theta$.

A. $-\dfrac{\sqrt{21}}{2}$

B. $\dfrac{\sqrt{21}}{5}$

* C. $-\dfrac{\sqrt{29}}{2}$

D. $\dfrac{\sqrt{29}}{5}$

Higher Mental Processes

A16

31. Point M $(-a,\ b)$ is in quadrant II and lies on the terminal arm of angle θ in standard position. Point N is the point of intersection of the terminal arm of angle θ and the unit circle centred at $(0,\ 0)$. Determine the *x*-coordinate of point N in terms of a and b.

* A. $\dfrac{-a}{\sqrt{a^2+b^2}}$

B. $\dfrac{-b}{\sqrt{a^2+b^2}}$

C. $\dfrac{a}{\sqrt{a^2+b^2}}$

D. $\dfrac{b}{\sqrt{a^2+b^2}}$

A17 ***sketch and analyze the graphs of sine, cosine and tangent functions, for***

- ***amplitude, if defined***
- ***period***
- ***domain and range***
- ***asymptotes, if any***
- ***behaviour under transformations***

Clarification: Graphs of the reciprocal trigonometric functions are analyzed in a similar manner to the graphs of the sine, cosine and tangent functions. Transformation on the graphs of reciprocal trigonometric functions are limited to horizontal and/or vertical expansions or compressions (i.e. no translations or reflections).

Knowledge 

A17

32. Determine the amplitude of $y = -3\cos 4x + 2$.

A. -4
B. -3
* C. 3
D. 4

Understanding 

A17

33. Determine the period of $y = \sin\frac{2\pi}{3}(x-6)$.

* A. 3

B. 6

C. $\frac{2\pi}{3}$

D. $\frac{4\pi}{3}$

Understanding 

A17

34. Determine the range of the function $y = 6\cos\frac{1}{2}(x-3)+4$.

A. $-6 \le y \le 6$

B. $1 \le y \le 7$

C. $-4 \le y \le 4$

* D. $-2 \le y \le 10$

Understanding 

A17

35. Which of the following lines is an asymptote for the graph of $y = \csc 2x$?

A. $x = 1$

B. $x = \frac{\pi}{4}$

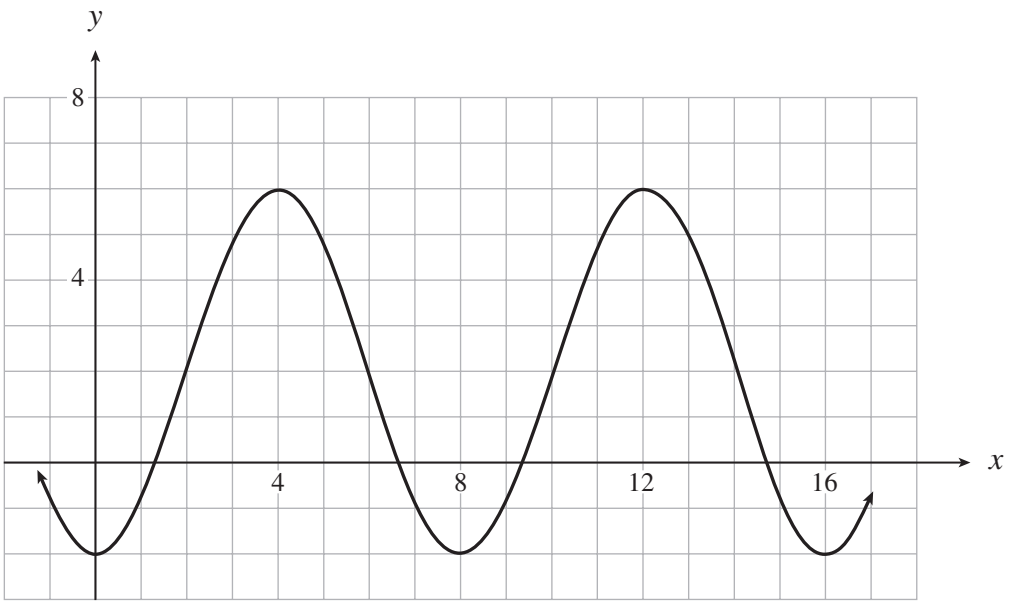
* C. $x = \frac{\pi}{2}$

D. $x = \frac{3\pi}{4}$

Understanding

A17

36. If the graph of the function shown below has the equation $y = a \sin b(x - c) + d$, determine the value of b. $(b > 0)$



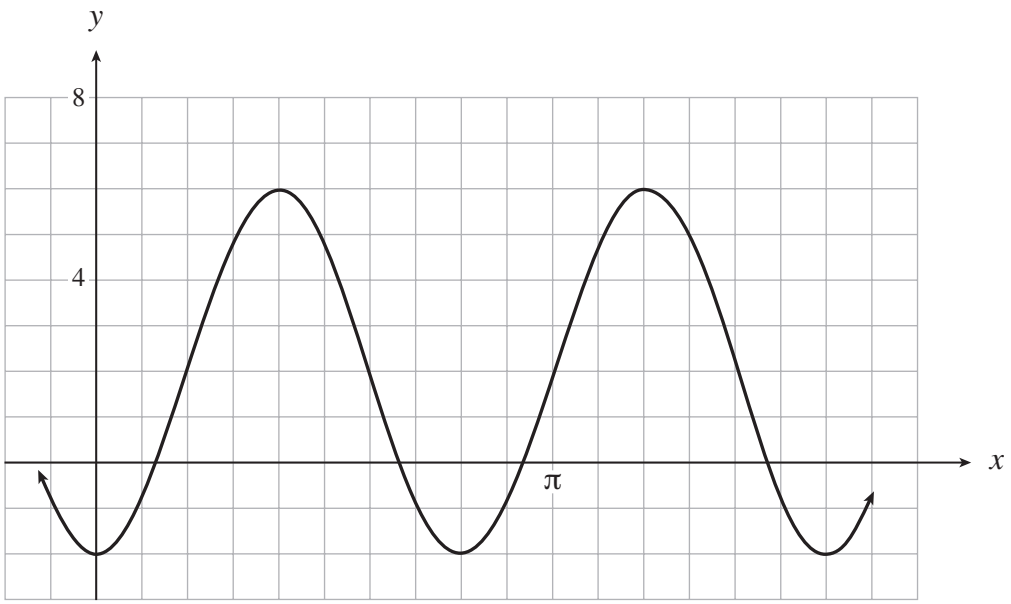
A. 4

B. 8

* C. $\frac{\pi}{4}$

D. $\frac{\pi}{8}$

37. If the graph of the function shown below has the equation $y = a \sin b(x - c) + d$, determine the value of b. $(b > 0)$



A. $\frac{5}{4}$

* B. $\frac{5}{2}$

C. $\frac{2\pi}{5}$

D. $\frac{4\pi}{5}$

Understanding

A17

38. State the phase shift of the function $y = -\cos\left(4x - \frac{\pi}{2}\right)$.

* A. $\frac{\pi}{8}$ to the right

B. $\frac{\pi}{8}$ to the left

C. $\frac{\pi}{2}$ to the right

D. $\frac{\pi}{2}$ to the left

Higher Mental Processes

A17

39. Determine the domain of $f(x) = \tan 2x$.

A. x = all real numbers

* B. x = all real numbers, $x \neq \frac{\pi}{4} + \frac{n\pi}{2}$, n is an integer

C. x = all real numbers, $x \neq \frac{\pi}{2} + n\pi$, n is an integer

D. x = all real numbers, $x \neq \pi + 2n\pi$, n is an integer

A18 ***use trigonometric functions to model and solve problems***

Understanding

A18

40. At a seaport, the depth of the water, d, in metres, at time t hours, during a certain day is given by:

$$d = 3.4 \sin 2\pi \frac{(t - 7.00)}{10.6} + 2.8$$

On that day, determine the depth of the water at 6:30 p.m.

A. 1.39 m
B. 1.81 m
C. 2.80 m
* D. 4.53 m

Note: *Students will need to express time in decimals of hours on a 24-hour clock.*

Higher Mental Processes 

A18

41. A wheel with radius 20 cm has its centre 30 cm above the ground. It rotates once every 15 seconds. Determine an equation for the height, h, above the ground of a point on the wheel at time, t seconds if this point has a maximum height at $t = 2$ seconds.

A. $h = 20 \cos \frac{2\pi}{15}(t + 2) + 30$

* B. $h = 20 \cos \frac{2\pi}{15}(t - 2) + 30$

C. $h = 30 \cos \frac{2\pi}{15}(t + 2) + 20$

D. $h = 30 \cos \frac{2\pi}{15}(t - 2) + 20$

42. A Ferris wheel with a diameter of 60 m rotates once every 48 seconds. At time $t = 0$, a rider is at his lowest height which is 2 m above the ground.

    a) Determine a sinusoidal equation that gives the height, h, of the rider above the ground as a function of the elapsed time, t, where h is in metres and t is seconds.

    b) Determine the time t when the rider will be 38 m above the ground for the first time after $t = 0$.

■ SOLUTION

Answers can vary. Some answers are:

a)

$$h = -30\cos\frac{\pi}{24}t + 32$$

$$h = 30\cos\frac{\pi}{24}(t - 24) + 32$$

$$h = 30\sin\frac{\pi}{24}(t - 12) + 32$$

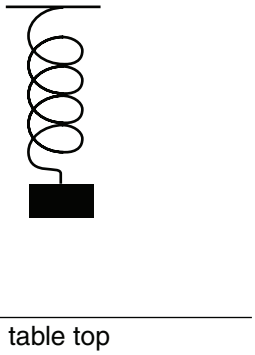
b)

$$38 = -30\cos\frac{\pi}{24}t + 32$$

$$t = 13.54 \text{ seconds}$$

Note: *This answer may be obtained using a graphing calculator.*

42. A mass is supported by a spring so that it rests 50 cm above a table top, as shown in the diagram below. The mass is pulled down to a height of 20 cm above the table top and released at time $t = 0$. It takes 0.8 seconds for the mass to reach a maximum height of 80 cm above the table top. As the mass moves up and down, its height h, in cm, above the table top, is approximated by a sinusoidal function of the elapsed time t, in seconds, for a short period of time.



Determine an equation for a sinusoidal function that gives h as a function of t.

■ SOLUTION

Answers can vary. Some answers are:

$$h = -30\cos\frac{2\pi}{1.6}t + 50$$

$$h = 30\cos\frac{2\pi}{1.6}(t - 0.8) + 50$$

$$h = 30\sin\frac{2\pi}{1.6}(t - 0.4) + 50$$

B SHAPE AND SPACE

Transformations

— Transformations

Clarification: Students need to be familiar with the term "invariant points" as points that are not altered by a transformation.

B1 ***describe how vertical and horizontal translations of functions affect graphs and their related equations:***

$$y = f(x - h)$$
$$y - k = f(x)$$

Knowledge

B1

1. If the graph of $2x + 3y = 5$ is translated 4 units up, determine an equation of the new graph.

   A. $2x + 3y = 1$

   B. $2x + 3y = 9$

   C. $2x + 3(y + 4) = 5$

   * D. $2x + 3(y - 4) = 5$

Understanding

B1

2. If $(a,\ b)$ is a point on the graph of $y = f(x)$, determine a point on the graph of $y = f(x - 2) + 3$.

   A. $(a - 2,\ b + 3)$

   B. $(a - 2,\ b - 3)$

   * C. $(a + 2,\ b + 3)$

   D. $(a + 2,\ b - 3)$

Understanding
B1

3. If the point $(2, -8)$ is on the graph of $y = f(x-3)+4$, what point must be on the graph of $y = f(x)$?

* A. $(-1, -12)$

B. $(-1, -4)$

C. $(5, -12)$

D. $(5, -4)$

B2 ***describe how compressions and expansions of functions affect graphs and their related equations:***

$$y = af(x)$$
$$y = f(kx)$$

Knowledge
B2

4. How is the graph of $y = 7^{3x}$ related to the graph of $y = 7^{x}$?

   A. The graph of $y = 7^{x}$ has been expanded vertically by a factor of 3.

   B. The graph of $y = 7^{x}$ has been compressed vertically by a factor of $\frac{1}{3}$.

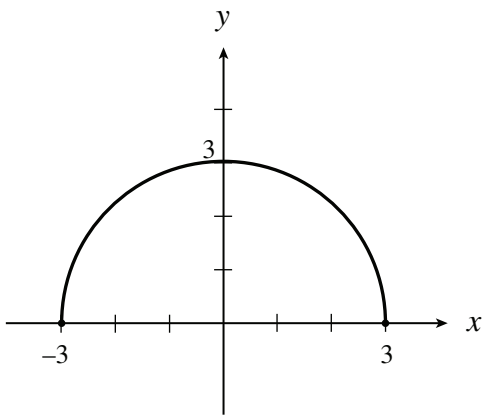
   C. The graph of $y = 7^{x}$ has been expanded horizontally by a factor of 3.

* D. The graph of $y = 7^{x}$ has been compressed horizontally by a factor of $\frac{1}{3}$.

Understanding

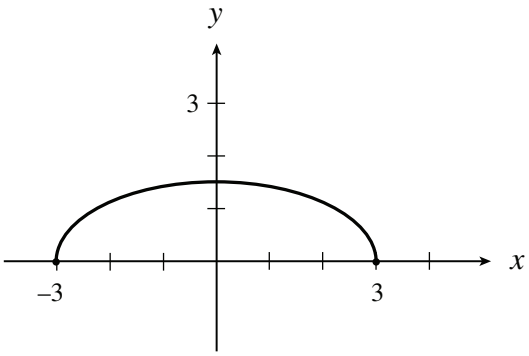
B2

5. The graph of $y = \sqrt{9 - x^2}$ is shown below.

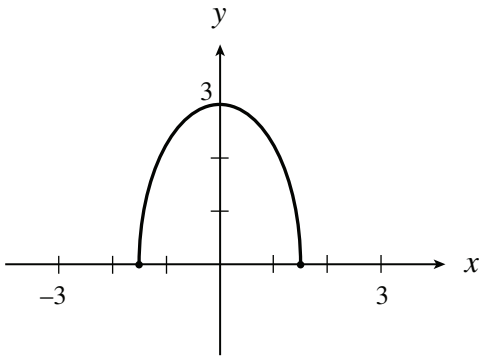


Which of the following graphs represents $2y = \sqrt{9 - x^2}$?

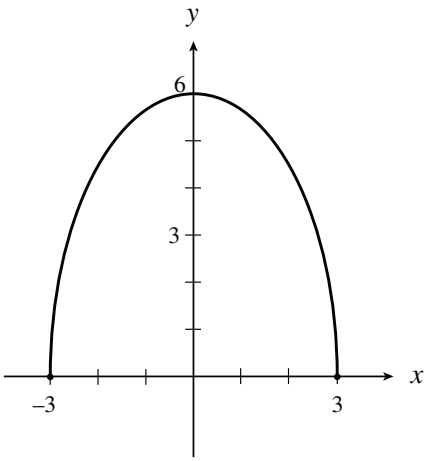
* A.



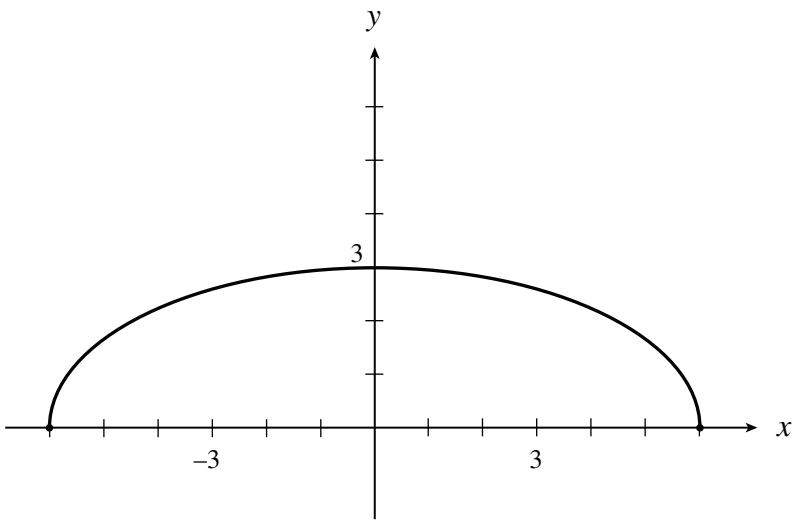
B.



C.



D.



B2, B3

6. If the graph of $x^2 + y^2 = 4$ is vertically compressed by a factor of $\frac{1}{5}$, then reflected in the *y*-axis, determine an equation for the new graph.

A. $x^2 + \frac{y^2}{25} = 4$

* B. $x^2 + 25y^2 = 4$

C. $-x^2 + 25y^2 = 4$

D. $-x^2 + \frac{y^2}{25} = 4$

B3 ***describe how reflections of functions in both axes and in the line $y = x$ affect graphs and their related equations:***

$y = f(-x)$ $y = -f(x)$ $y = f^{-1}(x)$

Knowledge
B3

7. The graph of $y = -f(x)$ is a reflection of the graph of $y = f(x)$ in

   A. the *y*-axis.
* B. the *x*-axis.
   C. the line $y = x$.
   D. the line $y = -x$.

Knowledge 
B3

8. What is the inverse of the relation $y = x^3$?

   A. $y = \frac{1}{x^3}$
* B. $x = y^3$
   C. $y = (-x)^3$
   D. $x = y^{\frac{1}{3}}$

Understanding
B3

9. The point $(6, -12)$ is on the graph of the function $y = f(x)$. Which point must be on the graph of the function $y = 3f(-x)$?

* A. $(-6, -36)$
B. $(6, 36)$
C. $(-6, -4)$
D. $(6, 4)$

Higher Mental Processes
B3

10. If $f(x) = \dfrac{2x}{x-1}$, determine the equation of $f^{-1}(x)$, the inverse of $f(x)$.

* A. $f^{-1}(x) = \dfrac{x}{x-2}$

B. $f^{-1}(x) = \dfrac{2x}{2x-1}$

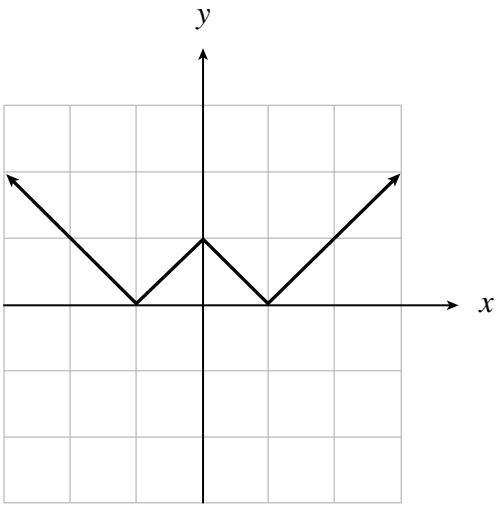
C. $f^{-1}(x) = \dfrac{x-1}{2x}$

D. $f^{-1}(x) = \dfrac{1}{x-2}$

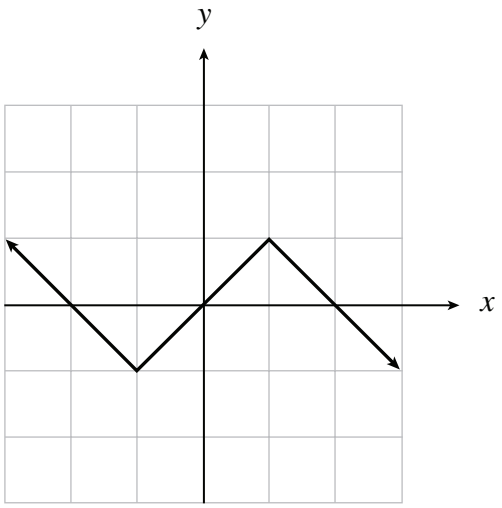
Higher Mental Processes
B3

11. For which graph of $y = f(x)$ would $f(-x) = -f(x)$?

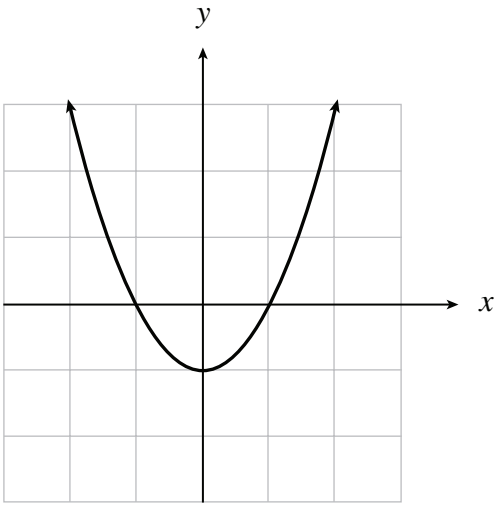
A.



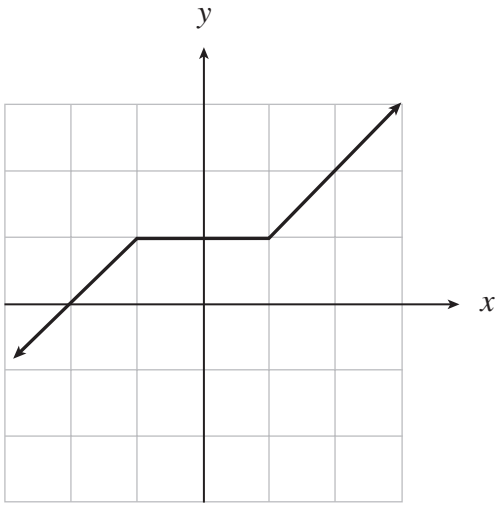
* B.



C.



D.



Understanding
B3

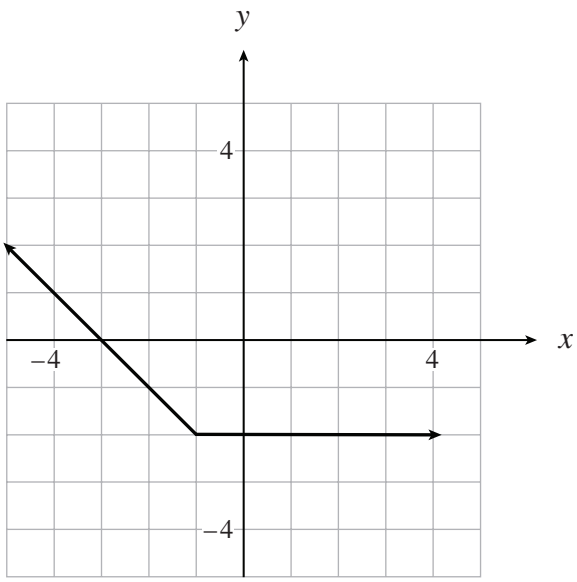
12. When the graph of $y = f(x)$ is transformed to the graph of $y = f(-x)$, on which line(s) will the invariant points lie?

A. $y = 0$
* B. $x = 0$
C. $y = x$
D. $y = 1,\ y = -1$

B4 ***using the graph and/or the equation of*** $f(x)$***, describe and sketch*** $\frac{1}{f(x)}$

Knowledge
B4

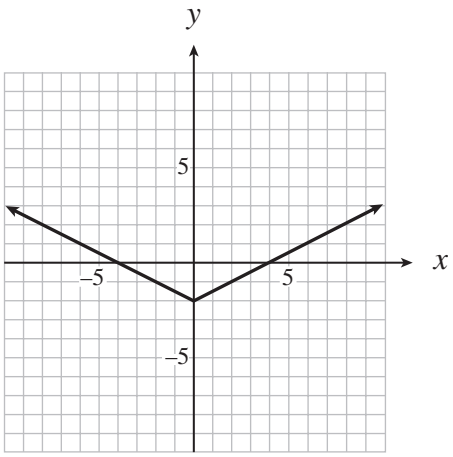
13. Given the graph of $y = f(x)$ below, determine an equation for an asymptote for the graph of $y = \frac{1}{f(x)}$.



A. $x = 3$
* B. $x = -3$
C. $y = -2$
D. $y = 2$

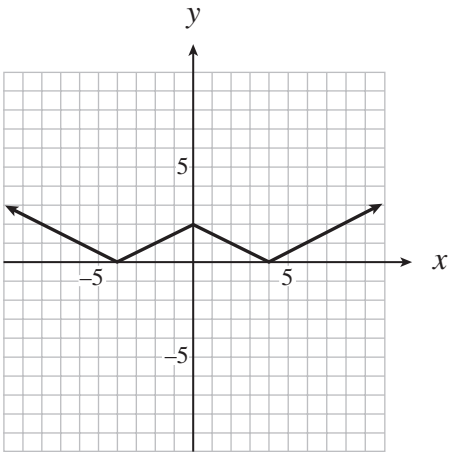
Understanding
B4

14. The graph of $y = f(x)$ is shown below.

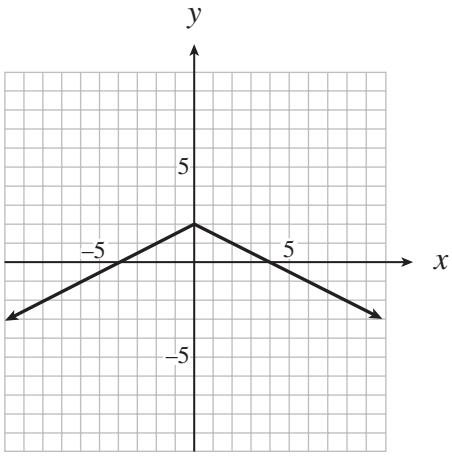


Which of the following graphs represents $y = \dfrac{1}{f(x)}$?

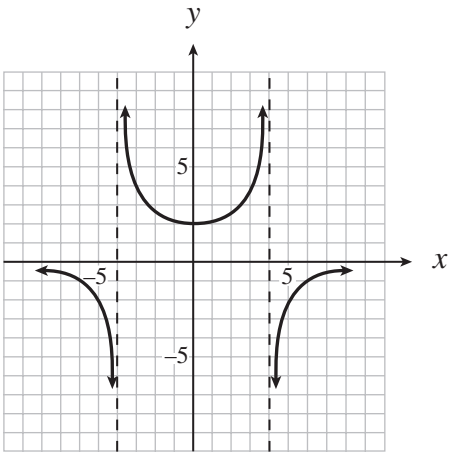
A.



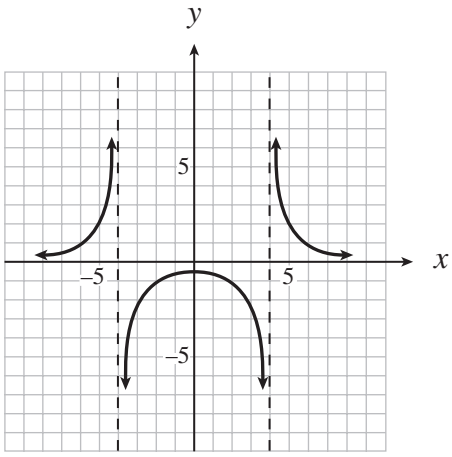
B.



C.



* D.



Higher Mental Processes
B4

15. If the range of $y = f(x)$ is $-1 \le y \le 2$, what is the range of $y = \frac{1}{f(x)}$?

A. $-1 \le y \le \frac{1}{2}$

B. $-1 \le y \le \frac{1}{2},\ y \ne 0$

* C. $y \ge \frac{1}{2}$ or $y \le -1$

D. $y \ge 2$ or $y \le -1$

Understanding
B4

16. The graph of $y = f(x)$ is transformed to the graph of $y = \frac{1}{f(x)}$. If the following points are on the graph of $y = f(x)$, which point would be invariant?

A. $(1,\ 2)$

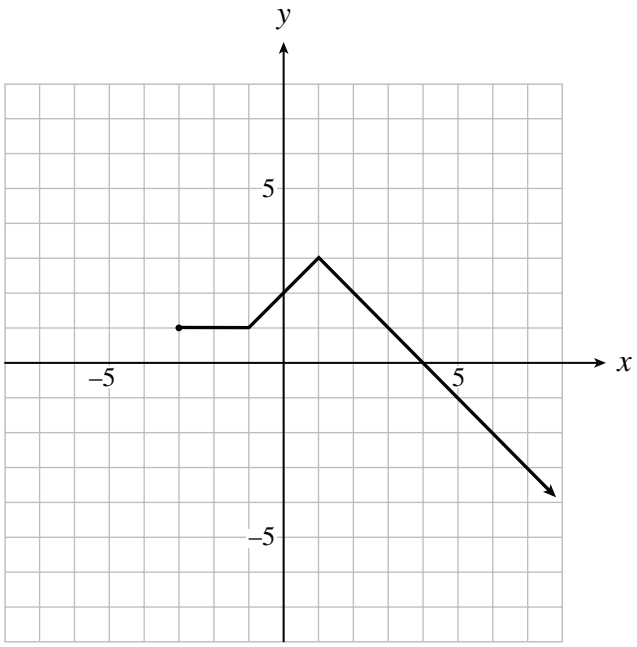
* B. $(2,\ 1)$

C. $(3,\ 0)$

D. $(0,\ 3)$

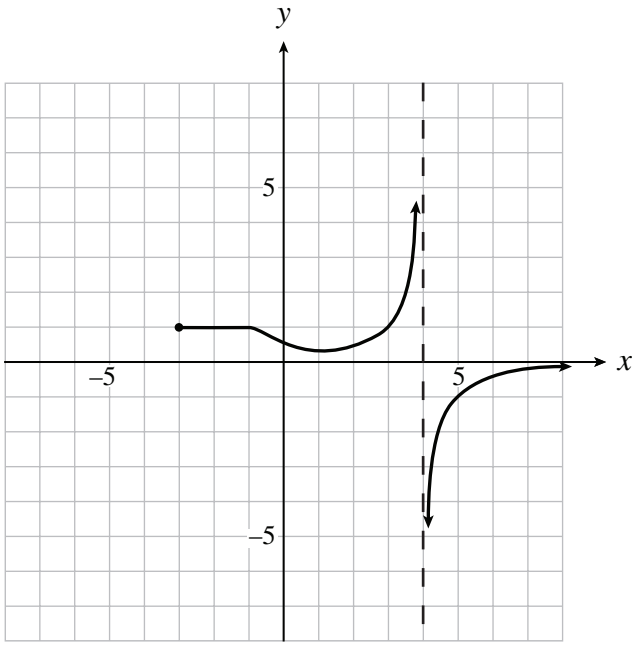
Understanding
B4

17. The graph of $y = f(x)$ is shown below.



On the grid provided, sketch the graph of $y = \frac{1}{f(x)}$.

■ SOLUTION

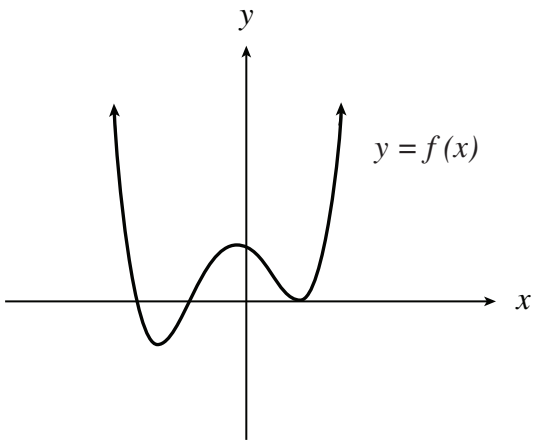


B5 ***using the graph and/or the equation of*** $f(x)$***, describe and sketch*** $|f(x)|$

Understanding

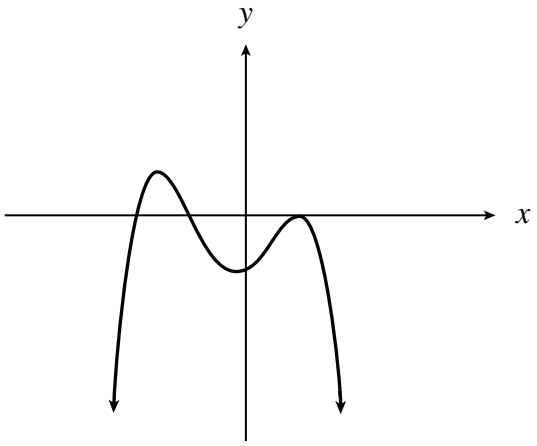
B5

18. The graph of the function $y = f(x)$ is shown below.

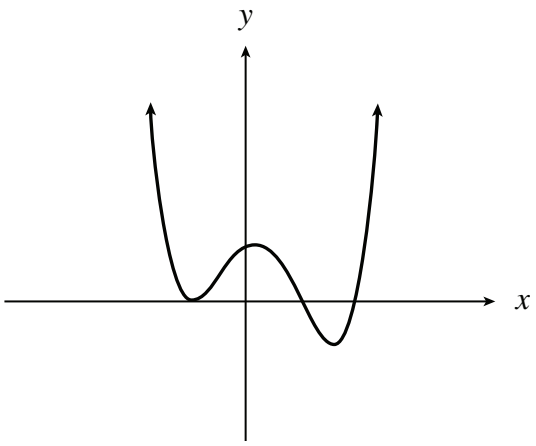


Which of the following is a graph of $y = |f(x)|$?

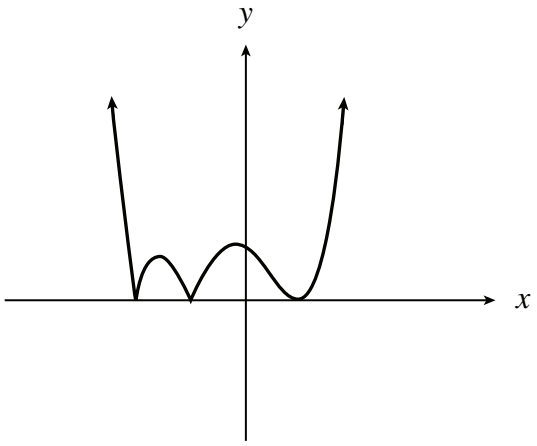
A.



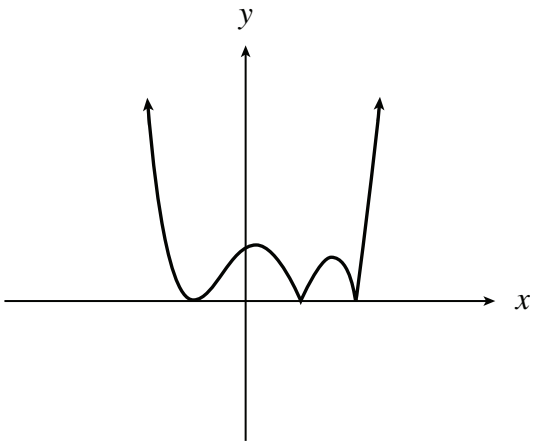
B.



* C.



D.

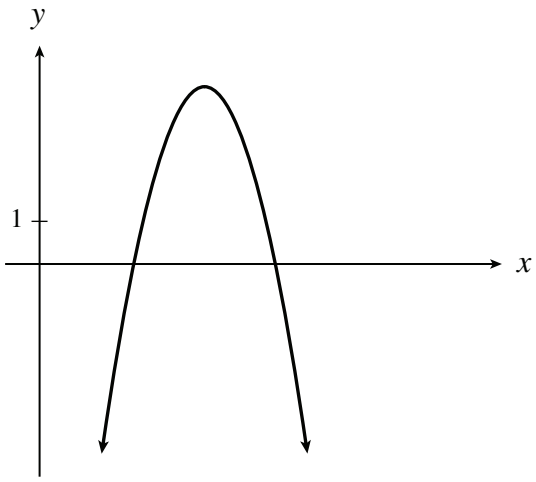


Understanding
B5

19. If the range of $y = f(x)$ is $-3 \le y \le 5$, what is the range of $y = |f(x)|$?

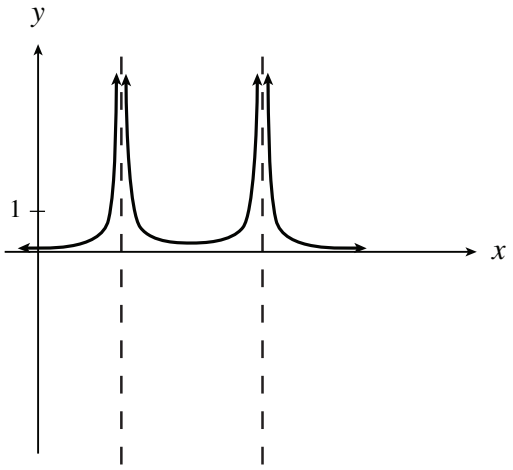
A. $-3 \le y \le 5$
B. $0 \le y \le 3$
* C. $0 \le y \le 5$
D. $3 \le y \le 5$

20. The graph of the function $y = f(x)$ is shown below.

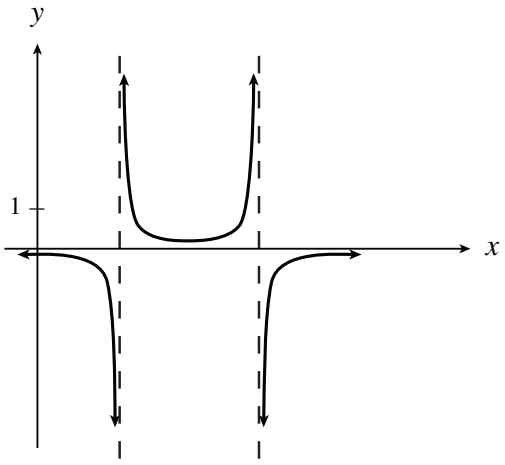


Which of the following is a graph of $y = \dfrac{1}{|f(x)|}$?

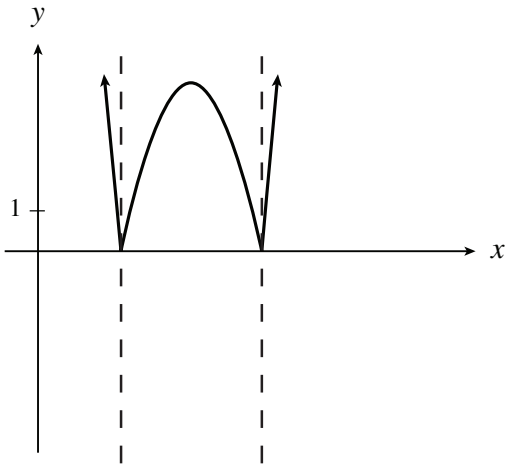
* A.



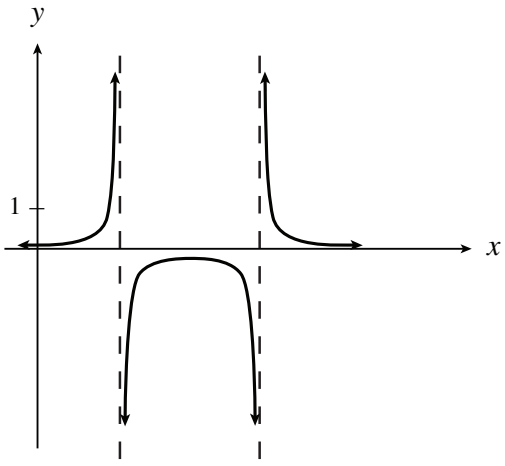
B.



C.



D.



B6 ***describe and perform single transformations and combinations of transformations on functions and relations***

Clarification: The absolute value of a function and the reciprocal value of a function may also be combined with transformations.

Knowledge

B6

21. Determine an equation that will cause the graph of $y = f(x)$ to expand vertically by a factor of 4 and shift 3 units up.

A. $y = \frac{1}{4} f(x) + 3$

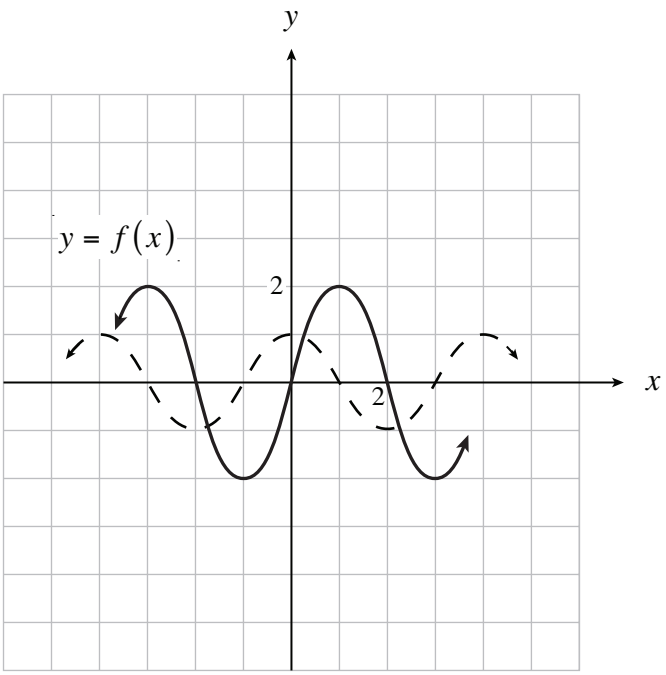
B. $y = \frac{1}{4} f(x) - 3$

* C. $y = 4f(x) + 3$

D. $y = 4f(x) - 3$

Understanding
B6

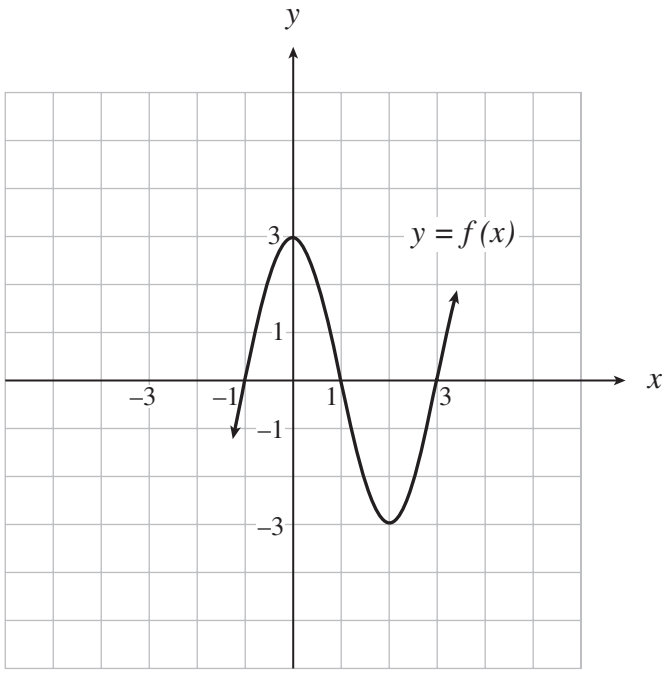
22. In the diagram below, $y = f(x)$ is graphed as a broken line.



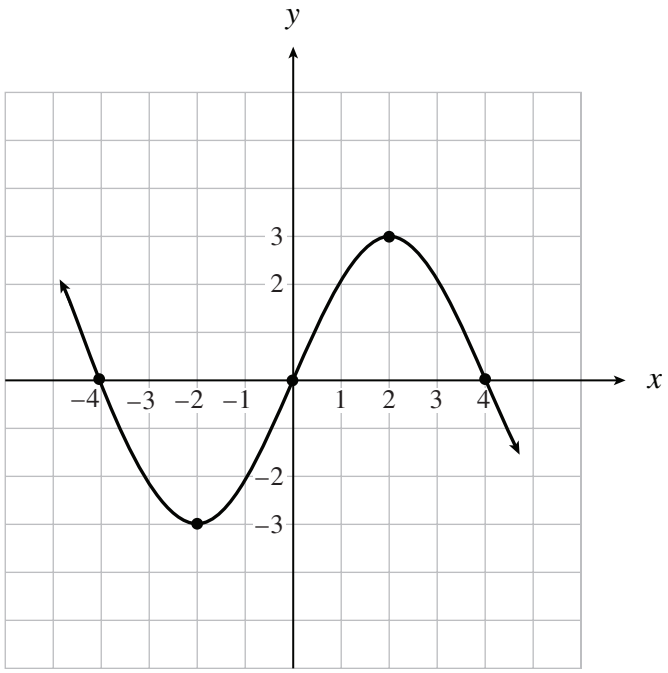
Which equation is defined by the solid line?

A. $y = 2f(x+1)$
B. $y = f(2x-1)$
C. $y = f(2x+1)$
* D. $y = 2f(x-1)$

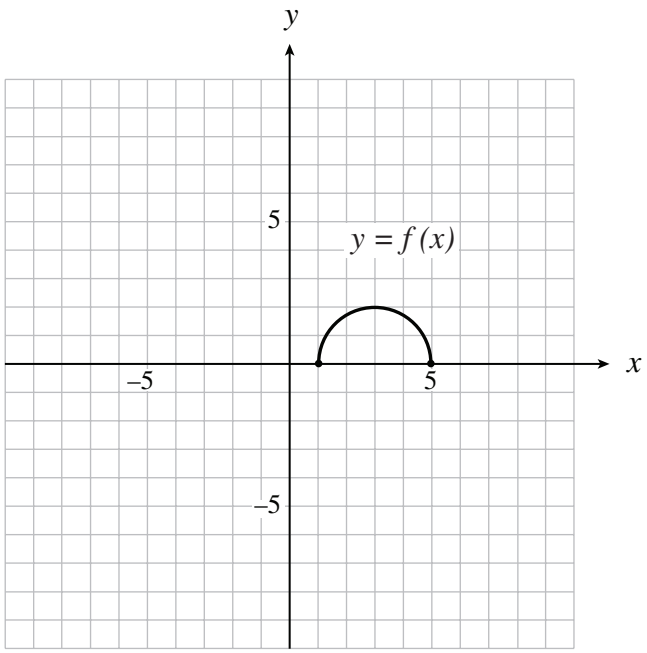
23. The graph of $y = f(x)$ is shown below. On the grid provided, sketch the graph of $y = -f\left(\frac{1}{2}(x+2)\right)$.

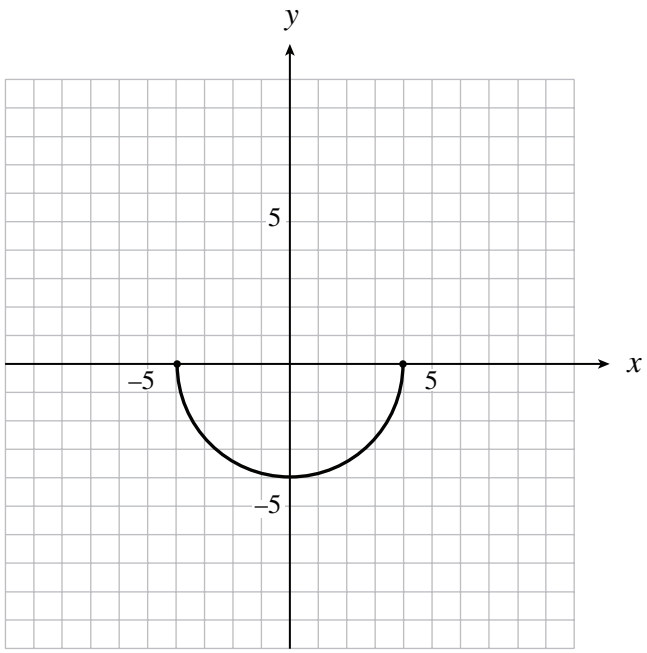


■ SOLUTION



24. The graph of $y = f(x)$ is shown below on the left. Which equation represents the graph shown on the right?





A. $y = -2f(2x+3)$

B. $y = -2f(2x+6)$

* C. $y = -2f\left(\frac{1}{2}x+3\right)$

D. $y = -2f\left(\frac{1}{2}x+6\right)$

25. If the point $(6, -2)$ is on the graph $y = f(x)$, which point must be on the graph of $y = \dfrac{1}{f(-x)+4}$?

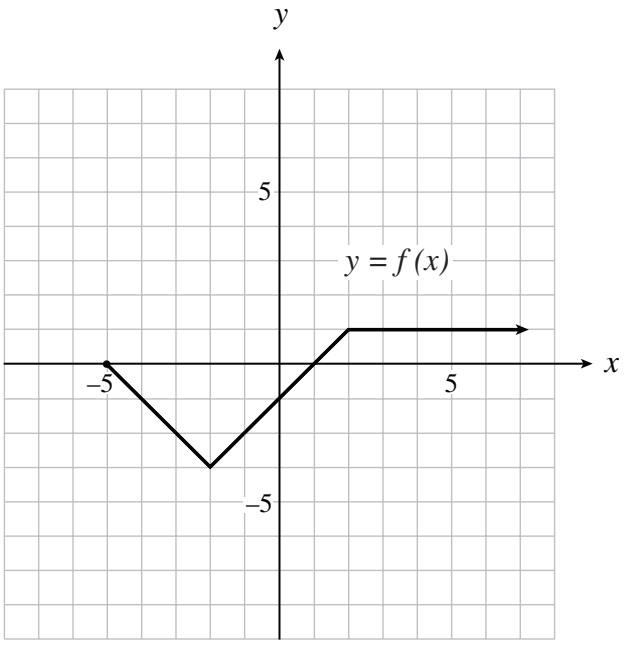
A. $\left(-10, -\dfrac{1}{2}\right)$

* B. $\left(-6, \dfrac{1}{2}\right)$

C. $\left(-6, \dfrac{7}{2}\right)$

D. $\left(-\dfrac{1}{6}, 2\right)$

26. The graph of $y = f(x)$ is shown below.

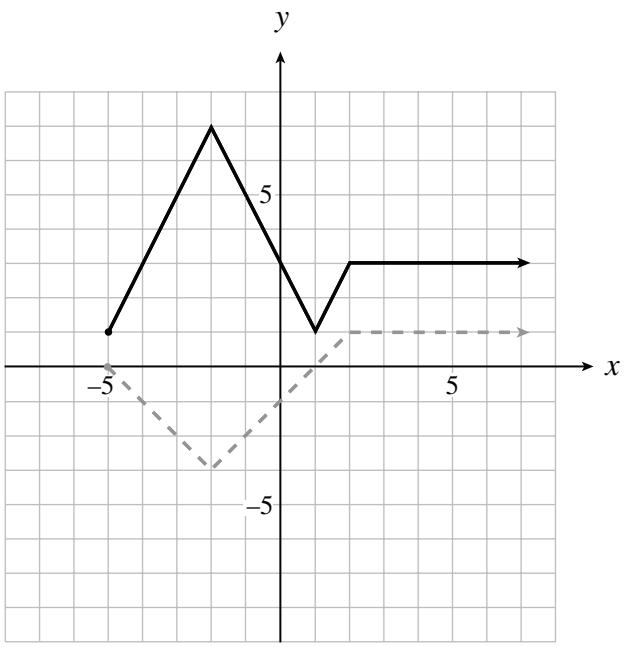


Understanding
B6

a) On the grid provided, sketch the graph of:

$$y = 2\left| f(x) \right| + 1$$

■ SOLUTION

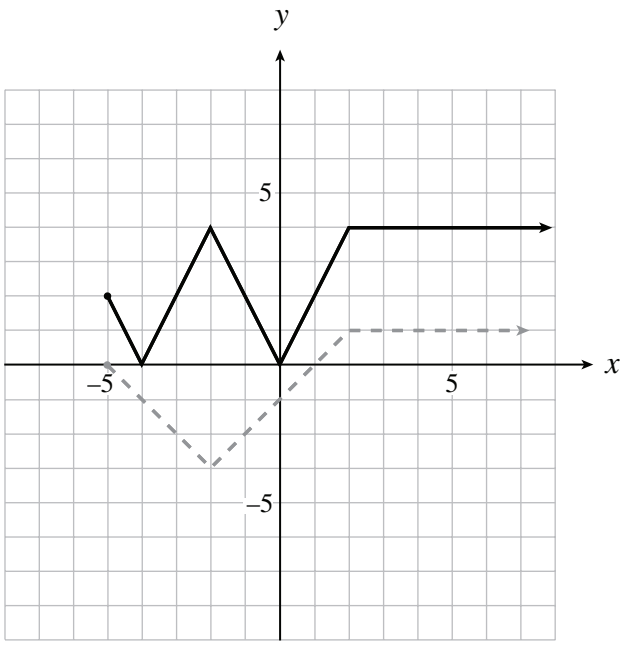


Understanding
B6

b) On the grid provided, sketch the graph of:

$$y = 2\left| f(x) + 1 \right|$$

■ SOLUTION



C STATISTICS AND PROBABILITY

Chance and Uncertainty

— Combinatorics

C1 ***use the fundamental counting principle to determine the number of different ways to perform multi-step operations***

Knowledge

C1

1. There are 45 multiple-choice questions on an exam with 4 possible answers for each question. How many different ways are there to complete the test?

   A. 45

   B. 45×4

   C. 45^4

   * D. 4^{45}

Understanding

C1

2. A breakfast special consists of choosing one item from each category in the following menu.

Juice:	apple, orange, grapefruit
Toast:	white, brown
Eggs:	scrambled, fried, poached
Beverage:	coffee, tea, milk

   How many different breakfast specials are possible?

   A. 11

   B. 48

   * C. 54

   D. 96

Understanding
C1

3. North American area codes are three digit numbers. Before 1995, area codes had the following restrictions: the first digit could not be 0 or 8, the second digit was either 0 or 1, and the third digit was any number from 1 through 9 inclusive. Under these rules, how many different area codes were possible?

   A. 112
   B. 120
* C. 144
   D. 504

C2 ***use factorial notation to determine different ways of arranging n distinct objects in a sequence***

Clarification: Factorial notation can also be used to determine different ways of arranging n objects some, of which are identical. Pathway problems can also provide an example of this use of factorial notation.

Understanding
C2

4. In a particular city, all of the streets run continuously north-south or east-west. The mayor lives 4 blocks east and 5 blocks north of city hall. Determine the number of different routes, 9 blocks in length, that the mayor can take to get to city hall.

   A. 20
* B. 126
   C. 3 024
   D. 15 120

Understanding 

C2

5. Simplify: $\frac{6!}{3!2!}$

   A. 1
   B. 20
* C. 60
   D. 120

Understanding

C2

6. A soccer team played 12 games in a season. They won 6 games, lost 4 games, and tied 2 games. In how many different orders could this have occurred?

   A. 576
* B. 13 860
   C. 9 979 200
   D. 31 933 440

Understanding

C2

7. Simplify the following expression without using the factorial symbol:

$$\frac{(n-2)!(n+1)!}{(n!)^2}$$

   A. $\frac{1}{n}$

   B. $\frac{1}{n-1}$

   C. $\frac{n-1}{n(n+1)}$

* D. $\frac{n+1}{n(n-1)}$

Higher Mental Processes 

C2

8. Solve for n: ${}_nC_3 = {}_nP_2$

   A. 6
* B. 8
   C. 1, 8
   D. 0, 1, 8

Understanding
C2

9. Solve: $\dfrac{n!}{(n-2)!3!} = 5$

■ SOLUTION

$$\frac{n(n-1)(n-2)!}{(n-2)!\,3!} = 5$$

$$\frac{n(n-1)}{6} = 5$$

$$n(n-1) = 30$$

$$n^2 - n - 30 = 0$$

$$(n+5)\ (n-6) = 0$$

$$n = -5 \quad n = 6$$

$\Downarrow$

reject

$\therefore\ n = 6$

Understanding
C2

10. There are 2 English books, 3 Chemistry books and 4 Mathematics books to be arranged on a shelf.

U a) If all the English books are identical, all the Chemistry books are identical and all the Mathematics books are identical, in how many different ways can they be arranged on the shelf?

U b) If the English books, Chemistry books and Mathematics books are all different, in how many ways can they be arranged on the shelf?

H c) If all the English books, Chemistry books and Mathematics books are different, in how many ways can they be arranged on the shelf if the Chemistry books have to be grouped together?

H d) If all the English books, Chemistry books and Mathematics books are different, in how many ways can they be arranged on the shelf if all the same subject books must be grouped together?

■ SOLUTION

a) $\frac{9!}{2!3!4!} = 1260$

b) $9! = 362\,880$

c) $7!3! = 30\,240$

d) $3!2!3!4! = 1728$

C2

11. Determine the number of different arrangements of all the letters in the word PARALLEL if

U a) there are no restrictions.

U b) the A's must be together.

U c) the first letter must be an A and the last letter must be an A.

H d) the first letter must be a vowel.

■ SOLUTION

a) $\frac{8!}{3!\,2!} = 3360$

b) $\frac{7!}{3!} = 840$

c) $\frac{6!}{3!} = 120$

d) *Case 1:* begins with A $\quad\quad$ *Case 2:* begins with E

$\frac{7!}{3!} = 840$ $\quad\quad$ $\frac{7!}{2!\,3!} = 420$

$840 + 420 = 1260$

C3 ***determine the number of permutations of n different objects taken r at a time, and use this to solve problems***

Knowledge

C3

12. A special combination lock that has 60 numbers on the dial works by turning it first to the right, then to the left, and then to the right, with 3 different selected numbers needed to open the lock. The selection of these 3 numbers is an example of

* A. a permutation.
B. a combination.
C. both a combination and a permutation.
D. neither a combination nor a permutation.

Understanding

C3

13. A soccer coach must choose 3 out of 10 players to kick tie-breaking penalty shots. Assuming the coach must designate the order of the 3 players, determine the number of different arrangements she has available.

* A. $\frac{10!}{7!}$

B. $\frac{10!}{3!}$

C. $\frac{10!}{3!\,7!}$

D. $\frac{10!}{3!\,3!\,4!}$

C4 ***determine the number of combinations of n different objects taken r at a time, and use this to solve problems***

Knowledge

C4

14. A bowl contains an apple, an orange, a plum and a banana. How many different pairs of fruit can be selected from the bowl?

    A. ${}_4P_2$
    B. ${}_2P_4$
* C. ${}_4C_2$
    D. ${}_2C_4$

Understanding

C4

15. In a standard deck of 52 cards, how many different 4-card hands are there that contain at most one heart?

    A. 91 403
    B. 118 807
    C. 188 474
* D. 201 058

Higher Mental Processes

C4

16. Assuming that at least one coin is used, how many different sums of money can be made from the following coins: a penny, a nickel, a dime, a quarter and a dollar?

    A. 16
* B. 31
    C. 32
    D. 120

Understanding
C4

17. There are five boys and six girls on a grad committee.

    a) In how many ways can a sub-committee of two boys and two girls be selected from the committee?

    b) In how many ways can a sub-committee of four people be selected if there must be at least one girl on the sub-committee?

■ SOLUTION

a)

number of ways two boys can be selected $= {}_5C_2$
number of ways two girls can be selected $= {}_6C_2$

$\therefore$ number of possible ways for a sub-committee of two boys and two girls to be selected

$= \left({}_5C_2\right)\left({}_6C_2\right) = (10)(15) = 150$ ways

b)

If there is at least one girl on the sub-committee, then we must consider the following cases:

1 girl, 3 boys $= \left({}_6C_1\right)\left({}_5C_3\right) = (6)(10) = 60$ ways

2 girls, 2 boys $= \left({}_6C_2\right)\left({}_5C_2\right) = (15)(10) = 150$ ways

3 girls, 1 boy $= \left({}_6C_3\right)\left({}_5C_1\right) = (20)(5) = 100$ ways

4 girls, 0 boys $= \left({}_6C_4\right)\left({}_5C_0\right) = (15)(1) = 15$ ways

$\therefore$ there are $60 + 150 + 100 + 15 = 325$ different ways in which a sub-committee of four students with at least one girl can be selected.

■ ALTERNATE SOLUTION

b)

Consider how many ways there could be a sub-committee of four students with **no** girls:

$$\left({}_6C_0\right)\left({}_5C_4\right) = 5$$

Total number of ways for a sub-committee of four students to be selected from eleven students

$$= {}_{11}C_4 = 330$$

$\therefore$ number of ways in which a sub-committee has **at least** one girl $= 330 - 5 = 325$

C5 ***solve problems, using the binomial theorem where the exponent n belongs to the set of natural numbers***

Clarification: Irregular pathway questions are considered an application of the binomial theorem.

Knowledge

C5

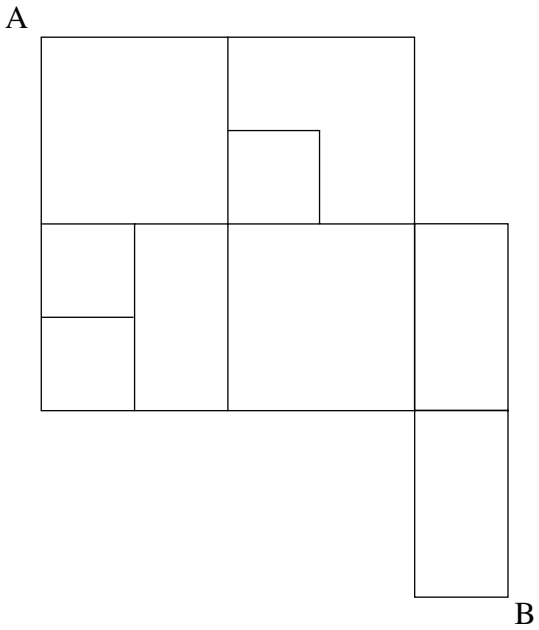
18. How many terms are in the expansion $\left(2x - \frac{1}{y}\right)^{10}$?

A. 9
B. 10
* C. 11
D. 12

Understanding

C5

19. Moving only to the right or down, how many different routes exist to get from point A to point B?



A. 19
* B. 22
C. 24
D. 37

Understanding
C5

20. The 10^{th} term of the expansion of $\left(x-\frac{1}{2}\right)^n$ is $-\frac{1001}{256}x^5$. Determine n.

A. 13
* B. 14
C. 15
D. 16

Understanding
C5

21. Determine the 8^{th} term of the expansion of $(2x-y)^{11}$.

* A. $-5280x^4y^7$
B. $-2640x^4y^7$
C. $1320x^3y^8$
D. $990x^3y^8$

Higher Mental Processes
C5

22. Which term in the expansion of $\left(\frac{1}{2x^2}-x^3\right)^{10}$ is a constant?

A. 4^{th}
* B. 5^{th}
C. 6^{th}
D. 11^{th}

Chance and Uncertainty

— Probability

C5 ***solve problems, using the binomial theorem where the exponent n belongs to the set of natural numbers***

Clarification: The binomial theorem can also be used to solve problems involving binomial ***probability*** *distributions*

Understanding

C5

23. The probability that a particular car will start on any morning is 0.9. Assuming that whether or not the car starts is independent from morning to morning, what is the probability that this car will start on at least 4 out of 5 mornings?

    (Answer accurate to at least 4 decimal places.)

■ SOLUTION

$p = 0.9 \qquad q = 0.1 \qquad n = 5$

$$\begin{aligned} \text{P (car starts on at least 4 out of 5 mornings)} &= \text{P (car starts on 4)} + \text{P (car starts on 5)} \\ &= \text{binompdf}(5,\ 0.9,\ 4) + \text{binompdf}(5,\ 0.9,\ 5) \\ &\approx 0.9185 \end{aligned}$$

or

$$P(x) = {}_nC_x\, p^x q^{n-x} \quad \Rightarrow \quad P(4) \text{ or } P(5) = {}_5C_4\,(0.9)^4\,(0.1)^1 + {}_5C_5\,(0.9)^5\,(0.1)^0$$
$$\approx 0.9185$$

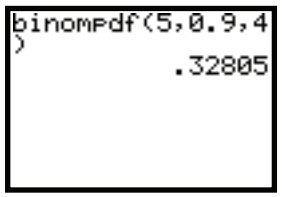
Note: The above experiment is a *binomial experiment* with 5 independent trials where the two outcomes are either: car starts (success) or car doesn't start (failure). Therefore, the binomial probability distribution can be used to solve the problem. In general, if the probability of success is p, the probability of failure is q, and the experiment is performed n times, then the probability of x successes is given by the formula: $P(x) = {}_nC_x\, p^x q^{n-x}$

Technology Note:

Many calculators have a built-in feature that determines the value for the binomial probability distribution. Screen images and key-strokes are provided below for reference.

Press [2nd] [DIST] [0] to obtain binompdf(, then enter n, p, x, where

n = the number of trials
p = the probability of "success"
x = number of "successes" desired



Understanding

C5

24. A multiple-choice test has 12 questions. Each question has 4 choices, only one of which is correct. If a student answers each question by guessing randomly, find the probability that the student gets:

U a) none of the questions correct.

U b) exactly 3 questions correct.

U c) at most 3 questions correct.

H d) at least 7 questions correct.

Answers accurate to at least 4 decimal places.

■ SOLUTION

$p = \frac{1}{4} \qquad q = \frac{3}{4} \qquad n = 12$

a) $P(\text{none of the questions correct}) = \text{binompdf}\left(12,\ \frac{1}{4},\ 0\right)$

$= 0.031676352$

≈ 0.0317

or

$P(\text{none of the questions correct}) = {}_{12}C_0 \left(\frac{1}{4}\right)^0 \left(\frac{3}{4}\right)^{12}$

$= 0.031676352$

≈ 0.0317

b) $P(3 \text{ questions correct}) = \text{binompdf}\left(12,\ \frac{1}{4},\ 3\right)$

$= 0.2581036091$

≈ 0.2681

or

$P(3 \text{ questions correct}) = {}_{12}C_3 \left(\frac{1}{4}\right)^3 \left(\frac{3}{4}\right)^9$

$= 0.2581036091$

≈ 0.2681

c) $P(\text{at most 3 questions correct}) = P(0, 1, 2, \text{or } 3 \text{ questions correct})$

$= \text{binomcdf}\left(12, \frac{1}{4}, 3\right)$

$= 0.6487786174$

≈ 0.6588

or

$P(\text{at most 3 questions correct}) = P(0, 1, 2, \text{or } 3 \text{ questions correct})$

$= {}_{12}C_0\left(\frac{1}{4}\right)^0\left(\frac{3}{4}\right)^{12} + {}_{12}C_1\left(\frac{1}{4}\right)^1\left(\frac{3}{4}\right)^{11} + {}_{12}C_2\left(\frac{1}{4}\right)^2\left(\frac{3}{4}\right)^{10} + {}_{12}C_3\left(\frac{1}{4}\right)^3\left(\frac{3}{4}\right)^9$

$= 0.6487786174$

≈ 0.6588

d) $P(\text{at least 7 questions correct}) = P(7, 8, 9, 10, 11, \text{or } 12 \text{ questions correct})$

$= 1 - P(0, 1, 2, 3, 4, 5 \text{ or } 6 \text{ questions correct})$

$= 1 - \text{binomcdf}\left(12, \frac{1}{4}, 6\right)$

$= 0.0142527818$

≈ 0.0143

or

$P(\text{at least 7 questions correct}) = P(7, 8, 9, 10, 11 \text{ or } 12 \text{ questions correct})$

$= {}_{12}C_7\left(\frac{1}{4}\right)^7\left(\frac{3}{4}\right)^5 + {}_{12}C_8\left(\frac{1}{4}\right)^8\left(\frac{3}{4}\right)^4 + {}_{12}C_9\left(\frac{1}{4}\right)^9\left(\frac{3}{4}\right)^3 + {}_{12}C_{10}\left(\frac{1}{4}\right)^{10}\left(\frac{3}{4}\right)^2 + {}_{12}C_{11}\left(\frac{1}{4}\right)^{11}\left(\frac{3}{4}\right)^1 + {}_{12}C_{12}\left(\frac{1}{4}\right)^{12}\left(\frac{3}{4}\right)^0$

$= 0.0142527818$

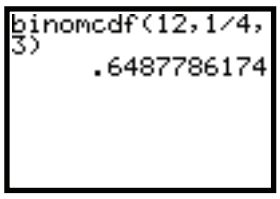
≈ 0.01

Technology Note:

Many calculators have a *binomial cumulative distribution function* binomcdf(.
Screen images and key-strokes are provided below for reference.

Press [2nd] [DIST] [A] to obtain binomcdf(, then enter n, p, x, where

n = the number of trials
p = the probability of "success"
x = the maximum number of "successes" desired



Note: *Use binomcdf if the* ***sum*** *of the probabilities of obtaining* $0, 1, 2, 3\ldots$ *or* x *successes is desired.*

C6 ***construct a sample space for up to three events***

Knowledge

C6

25. For each probability experiment, the set of all possible outcomes is called

    A. odds.
    B. event.
    C. element.
    * D. sample space.

Understanding

C6

26. A game begins with two cards being dealt from a standard deck of 52 cards. To win this game, the next card dealt must be the same as either of these first two cards, or fall between them. If the first two cards are a 3 and a 10, what is the probability of winning this game?

    * A. $\frac{30}{50}$

    B. $\frac{32}{50}$

    C. $\frac{30}{52}$

    D. $\frac{32}{52}$

27. One of two cards is black on one side and white on the other side. The second card is black on both sides. One card is selected at random and the side facing up is black. What is the probability that the other side of the card is white?

A. $\frac{1}{4}$

* B. $\frac{1}{3}$

C. $\frac{1}{2}$

D. $\frac{2}{3}$

C7 *classify events as independent or dependent*

Knowledge
C7

28. Which of the following pair of events is dependent?

* A. Two cards are selected from a well-shuffled deck of cards and the experiment is carried out without replacement. The first event is drawing a jack. The second event is drawing another jack.

B. Two cards are selected from a well-shuffled deck of cards and the experiment is carried out with replacement. The first event is drawing an ace of hearts. The second event is drawing a black 5.

C. A fair die is rolled and a fair coin is tossed. The first event is rolling an odd number on the die. The second event is obtaining a tail on a flip of the coin.

D. A fair coin is tossed twice. The first event is obtaining a head on the first flip of the coin. The second event is obtaining a head on the second flip of the coin.

C8 ***solve problems, using the probabilities of mutually exclusive and complementary events***

Understanding
C8

29. Two fair dice are rolled. A sample space is provided below.

Second Die

First Die	**1**	**2**	**3**	**4**	**5**	**6**
1	(1, 1)	(1, 2)	(1, 3)	(1, 4)	(1, 5)	(1, 6)
2	(2, 1)	(2, 2)	(2, 3)	(2, 4)	(2, 5)	(2, 6)
3	(3, 1)	(3, 2)	(3, 3)	(3, 4)	(3, 5)	(3, 6)
4	(4, 1)	(4, 2)	(4, 3)	(4, 4)	(4, 5)	(4, 6)
5	(5, 1)	(5, 2)	(5, 3)	(5, 4)	(5, 5)	(5, 6)
6	(6, 1)	(6, 2)	(6, 3)	(6, 4)	(6, 5)	(6, 6)

Consider the following events:

A: The sum of the two dice is 5.
B: The first die rolled is a 1.
C: The second die rolled is a 4.
D: The product of the two dice is 6.

Which of the two events given above are mutually exclusive?

A. A and B
B. A and C
C. B and C
* D. C and D

Understanding
C8

30. A summary of a recent survey is shown below:

60% liked hamburgers
70% liked pizza
40% liked both

What percentage liked neither?

* A. 10%
B. 20%
C. 30%
D. 40%

Understanding
C8

31. What is the probability of drawing a heart or a face card in a single random draw from a standard deck of 52 cards?

		Clubs	Diamonds	Hearts	Spades
Face cards	**King**	♣	♦	♥	♠
	Queen	♣	♦	♥	♠
	Jack	♣	♦	♥	♠
	10	♣	♦	♥	♠
	9	♣	♦	♥	♠
	8	♣	♦	♥	♠
	7	♣	♦	♥	♠
	6	♣	♦	♥	♠
	5	♣	♦	♥	♠
	4	♣	♦	♥	♠
	3	♣	♦	♥	♠
	2	♣	♦	♥	♠
	Ace	♣	♦	♥	♠

A. $\frac{13}{52}$

B. $\frac{12}{52}$

* C. $\frac{22}{52}$

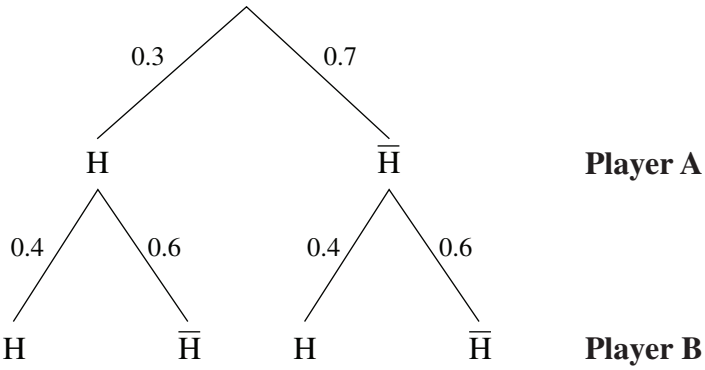
D. $\frac{25}{52}$

Note: This diagram is provided as an instructional tool, and may not be provided on an examination.

32. Two dart players each throw independently one dart at a target. The probability of each player hitting the bulls-eye is 0.3 and 0.4 respectively. What is the probability that at least one of them will hit the bulls-eye?

■ SOLUTION

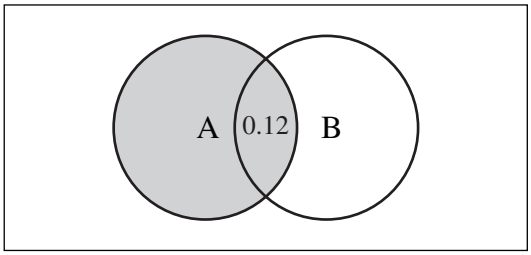
$$P(A \text{ or } B) = P(A) + P(B) - P(A \text{ and } B)$$
$$= 0.3 + 0.4 - (0.3)(0.4)$$
$$= 0.58$$



$$P(\text{at least 1}) = 0.3 \times 0.4 + 0.3 \times 0.6 + 0.7 \times 0.4$$
$$= 0.58$$

or $P(A \text{ and } B) = 0.3 \times 0.4 = 0.12$

$$P(A \text{ or } B) = P(A) + P(B) - P(A \text{ and B})$$
$$= 0.3 + 0.4 - 0.12$$
$$= 0.58$$



or

$$1 - (0.7)(0.6) = 0.58$$

33. If one of the 19 equally likely outcomes in the sample space **S** is randomly selected, find the probability that:

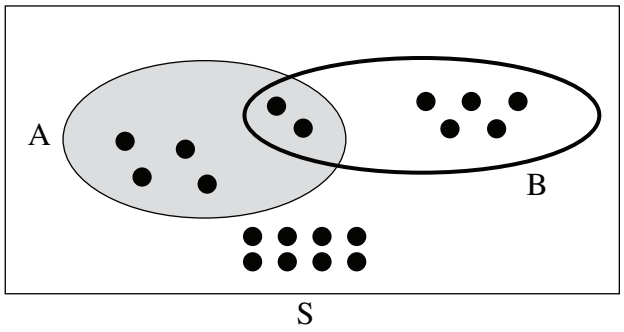
K a) both **A** and **B** occur.

U b) **A** but not **B** occurs.

U c) neither **A** nor **B** occurs.

U d) at least one of **A** or **B** occurs.

H e) at most one of **A** or **B** occurs.



■ SOLUTION

a) $P(A \text{ and } B) = \frac{2}{19}$

b) $P(A \text{ and } \bar{B}) = \frac{4}{19}$

c) $P(\bar{A} \text{ and } \bar{B}) = \frac{8}{19}$

d) $P(A \text{ or } B) = \frac{11}{19}$

e) $P(A \text{ or } B) - P(A \text{ and } B) = \frac{9}{19}$

C9 ***determine the conditional probability of two events***

Understanding

C9

34. Each of the 11 letters from the word MATHEMATICS is placed on a separate card. A card is drawn and not replaced. A second card is drawn. What is the probability that the 2 cards chosen are both vowels?

A. $\frac{1}{20}$

B. $\frac{1}{10}$

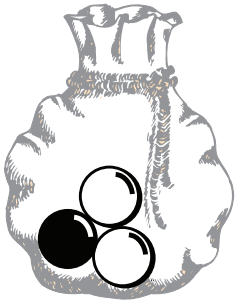
* C. $\frac{6}{55}$

D. $\frac{16}{121}$

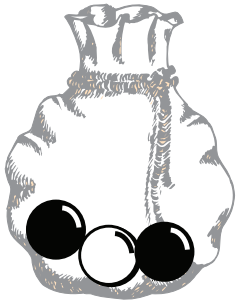
Higher Mental Processes

C9

35. Bag A contains 1 black and 2 white marbles, and Bag B contains 1 white and 2 black marbles. A marble is randomly chosen from Bag A and placed in Bag B. A marble is then randomly chosen from Bag B. Determine the probability that the marble selected from Bag B is white.



Bag A



Bag B

A. $\frac{5}{24}$

B. $\frac{1}{3}$

* C. $\frac{5}{12}$

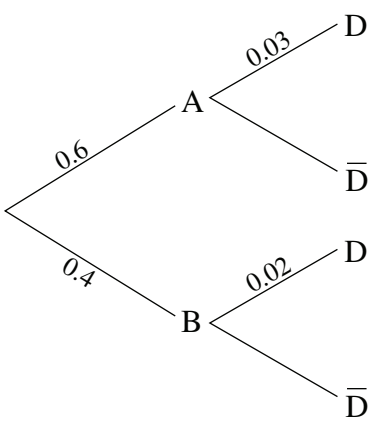
D. $\frac{1}{2}$

36. Machine A produces 60% of a product while Machine B produces 40%. It is known that 3% of the production from Machine A is defective, while 2% from Machine B is defective. If a defective product is selected, what is the probability that it was produced by Machine B?

(Answer accurate to at least 4 decimal places.)

■ SOLUTION

Let A = product from Machine A
B = product from Machine B
D = product is defective



$\rightarrow (0.6)(0.03) = 0.018$

$\rightarrow (0.4)(0.02) = 0.008$

$\therefore P(\text{defective product comes from B}) = \dfrac{0.008}{0.008 + 0.018} = 0.31$

or

$\dfrac{4}{13}$

■ ALTERNATE SOLUTION

$$P(\text{B} \mid \text{D}) = \frac{P(\text{B and D})}{P(\text{D})}$$

$$= \frac{0.4 \times (0.02)}{0.6(0.03) + 0.4(0.02)}$$

$$= \frac{4}{13}$$

or

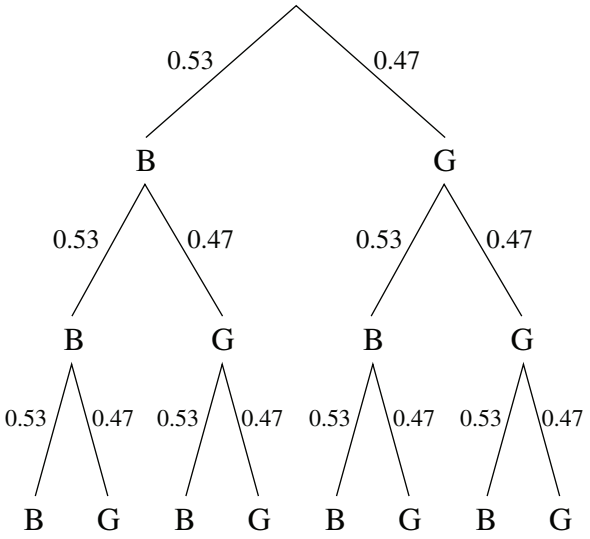
$$\approx 0.3077$$

Understanding
C9

37. It is known that 53% of graduating students are boys. Three grads are chosen at random. Given that at least two of the three grads are boys, determine the probability that all three of the grads are boys.

    (Answer accurate to at least 4 decimal places.)

■ SOLUTION



OR

$$P(\text{3 boys} \mid \text{at least 2 boys})$$

$$= \frac{P(\text{3 boys and at least 2 boys})}{P(\text{at least 2 boys})}$$

$$= \frac{0.53^3}{3(0.53)^2(0.47)+(0.53)^3}$$

$$\approx 0.2732$$

$$P(\text{3 boys} \mid \text{at least 2 boys})$$

$$= \frac{P(\text{3 boys and at least 2 boys})}{P(\text{at least 2 boys})}$$

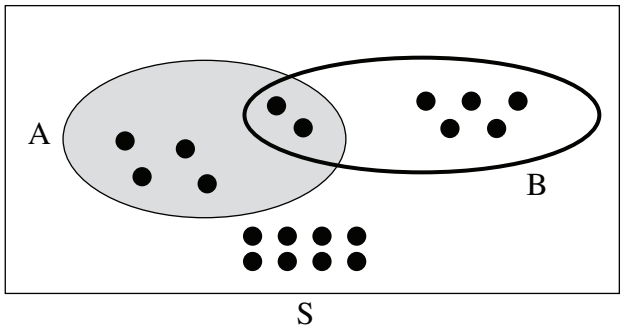
$$= \frac{\text{binompdf}(3,\ 0.53,\ 3)}{1-\text{binomcdf}(3,\ 0.53,\ 1)}$$

$$\approx 0.2732$$

38. If one of the 19 equally likely outcomes in the sample space **S** is randomly selected, find the probability that:

U a) **A** occurs given that **B** has occurred.

U b) **B** occurs given that **A** has occurred.



■ SOLUTION

a) $P(A \mid B) = \frac{2}{7}$

b) $P(B \mid A) = \frac{2}{6}$

Understanding
C9

39. A new test for a certain disease is found to be 98% accurate. This means that the outcome of the test is correct 98% of the time. If it is estimated that 1.2% of the population in a certain province has this disease, then determine:

    a) the probability that a randomly selected person from the province will test positive for the disease.

    b) the probability that a randomly selected person from the province has the disease given that the person tested positive.

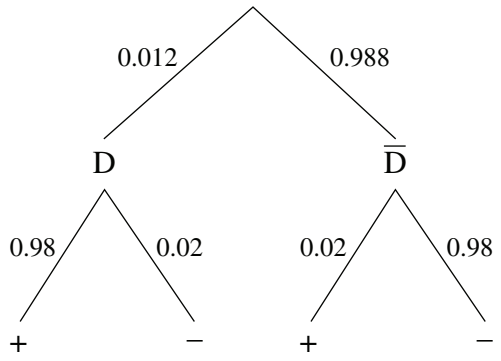
    (Answer accurate to at least 4 decimal places.)

■ SOLUTION

a)

$$P(+) = 0.012 \times 0.98 + 0.988 \times 0.02$$

$$\approx 0.0315$$



b)

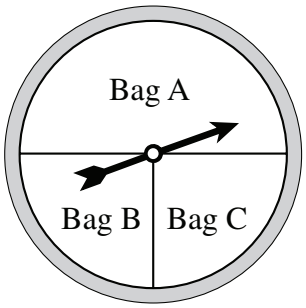
$$P(D \mid +) = \frac{P(D \text{ and } +)}{P(+)}$$

$$= \frac{0.012 \times 0.98}{0.03152}$$

$$= \frac{0.01176}{0.03152}$$

$$\approx 0.3731$$

40. The pointer is spun to determine a bag, and a marble is then randomly chosen from the selected bag.

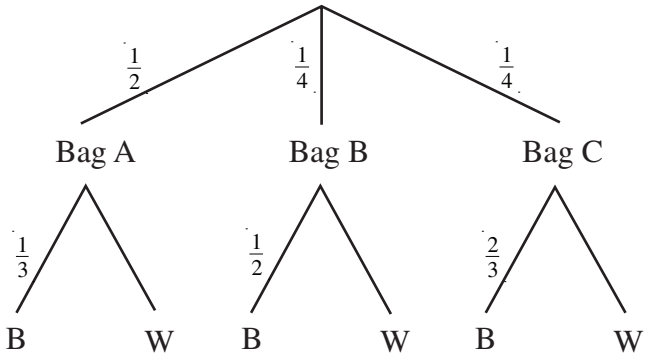


U a) What is the probability that the chosen marble is black?

H b) If the chosen marble is black, what is the probability that another randomly chosen marble from the same bag will also be black?

■ SOLUTION

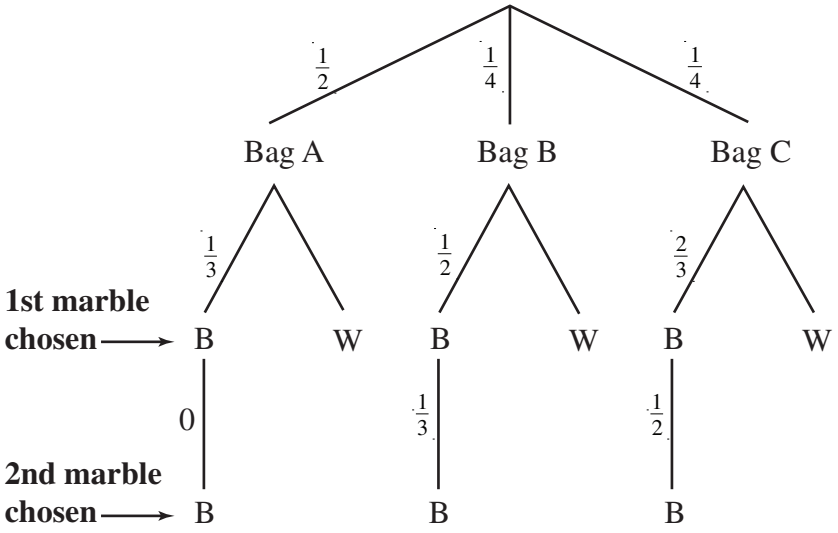
a) Let B = black marble chosen W = white marble chosen



$$P(B) = \left(\frac{1}{2} \times \frac{1}{3}\right) + \left(\frac{1}{4} \times \frac{1}{2}\right) + \left(\frac{1}{4} \times \frac{2}{3}\right)$$

$$= \frac{11}{24}$$

b) Consider one more level of the tree.



$$\frac{P(\text{second marble B and first marble B})}{P(\text{first marble B})}$$

$$P(\text{second marble B} \mid \text{first marble B}) = \frac{\left(\frac{1}{2} \times \frac{1}{3} \times 0\right) + \left(\frac{1}{4} \times \frac{1}{2} \times \frac{1}{3}\right) + \left(\frac{1}{4} \times \frac{2}{3} \times \frac{1}{2}\right)}{\frac{11}{24}}$$

$$= \frac{3}{11}$$

C10 ***solve probability problems involving permutations, combinations, and conditional probability***

Understanding
C10

41. Six people are randomly selected from a group of 8 males and 10 females to form a committee. Determine the probability that exactly 4 males are selected for this committee.

    A. 0.01
    B. 0.10
* C. 0.17
    D. 0.32

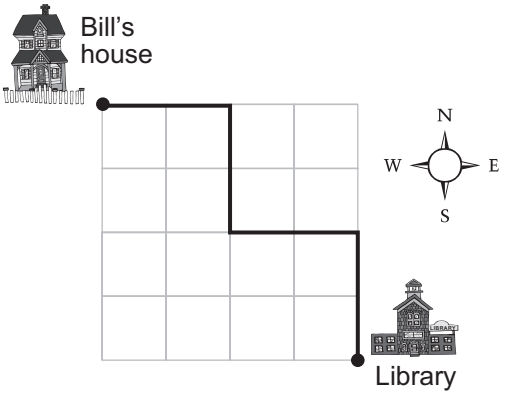
Understanding
C10

42. If 5 cards are dealt from a standard deck of 52 cards, determine the probability of obtaining 3 red cards and 2 black face cards.

    A. 0.0010
* B. 0.0150
    C. 0.0660
    D. 0.3251

Understanding
C10

43. Bill is walking from his house to the library. If Bill only walks south or east, determine the probability that he will select the route indicated in the diagram below. Assume that all routes have an equal chance of being chosen.



A. $\frac{1}{20}$

B. $\frac{1}{35}$

C. $\frac{1}{55}$

* D. $\frac{1}{70}$

Higher Mental Processes
C10

44. Five balls are randomly drawn without replacement from a bag containing 4 red balls and 6 black balls. What is the probability that at least 3 red balls will be drawn?

A. 0.0238
B. 0.2381
* C. 0.2619
D. 0.7381