

Lesson 3: Kirchoff's Laws

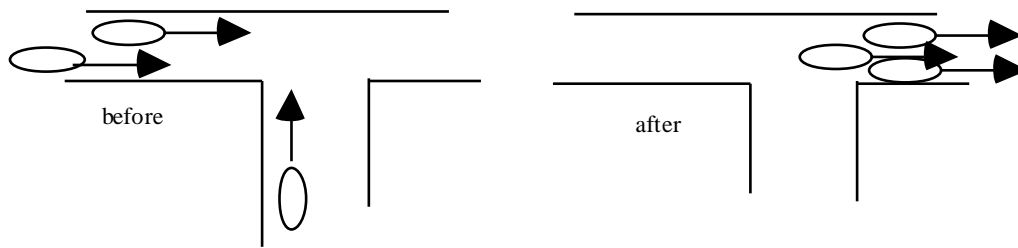
•Kirchoff's Current Law (KCL) is basically conservation of charge:

KCL states that at a junction between wires, current flow splits or combines and continues to flow 'downhill'. When the current splits or combines, no charge is lost, so that the total flow into the junction equals the total flow out of the junction.

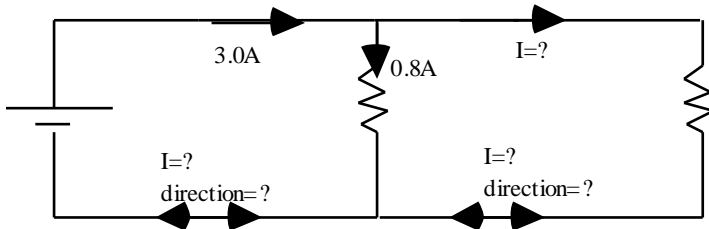
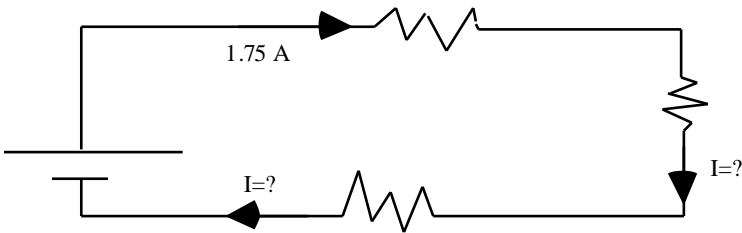
$$I_{in} = \text{current into/towards junction} = \text{Amp}$$

$$I_{out} = \text{current out of/away from junction} = \text{Amp}$$

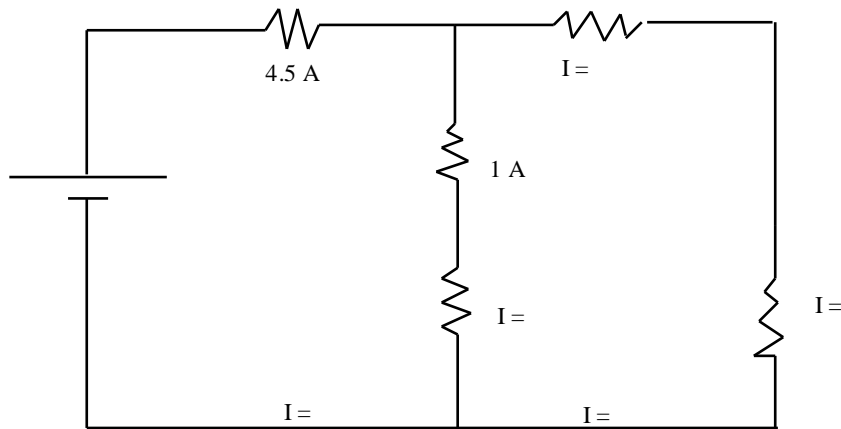
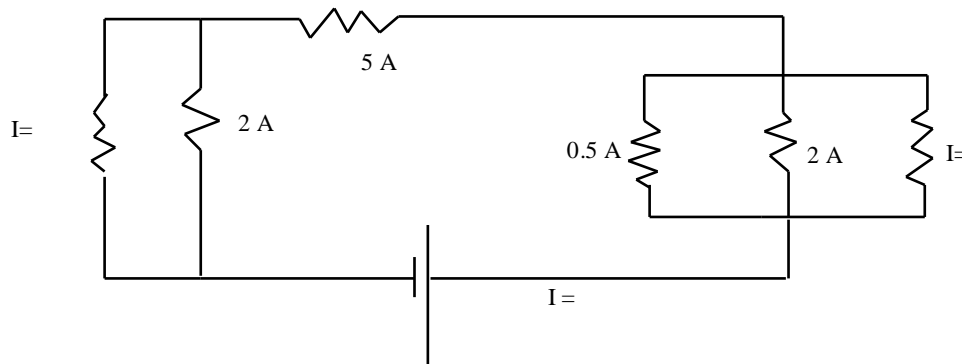
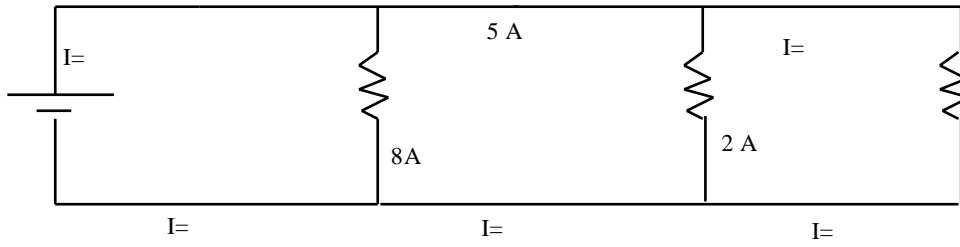
•In terms of the ski hill analogy, KCL simply means that if three skiers converge on an intersection from two different paths, then three skiers must leave the intersection still going downhill.



•example 1: Find the unknown currents

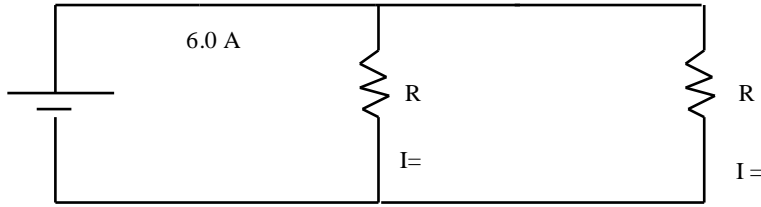


•example 2: Show the direction of each current, and find unknown currents

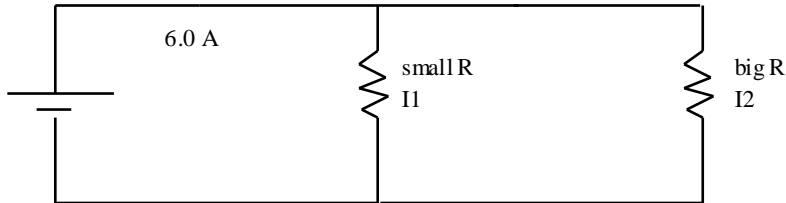


•The amount of flow along a particular path from a junction depends on the resistance of that path.

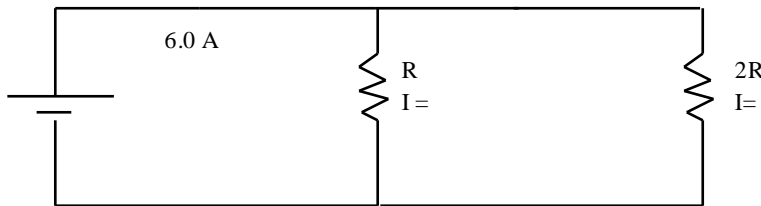
•example 3: a) If the resistance of each path leading away from a junction is the same, what can be said about the splitting of the current? For instance, find the unknown currents below.



b) if one path has more resistance then what can we say about the splitting of the current? For instance compare the unknown currents I_1 and I_2 below



c) if the resistors are simple ratios of each other, we can relate the currents via the same ratios. For instance, find the unknown currents below. (note the second resistor is twice the first)



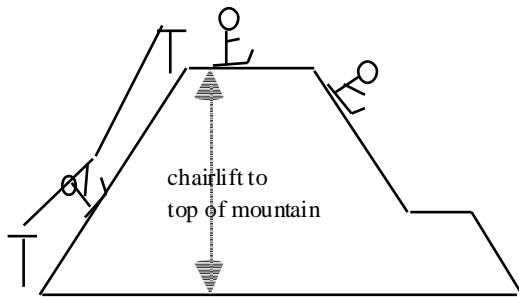
•Kirchhoff's Voltage Law (KVL) is basically conservation of energy:

KVL states that as charges flow around the circuit, they experience voltage gains in the cell and voltage drops across the resistors. For every path through the circuit the voltage gains equal the voltage losses.

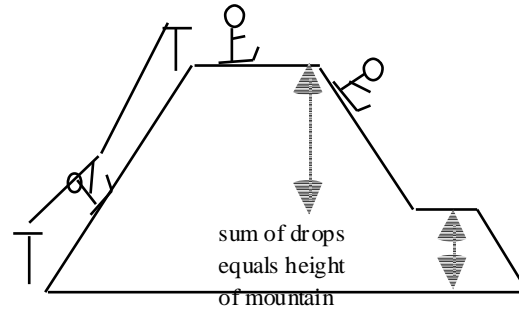
$$V_{\text{gains}} = \text{gains on voltage in a loop} = \text{volt}$$

$$V_{\text{losses}} = \text{losses of voltage in a loop} = \text{volt}$$

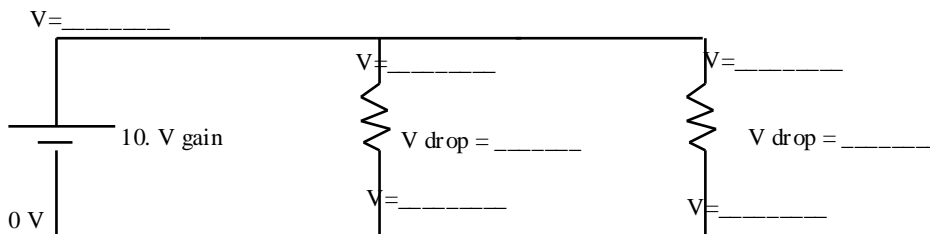
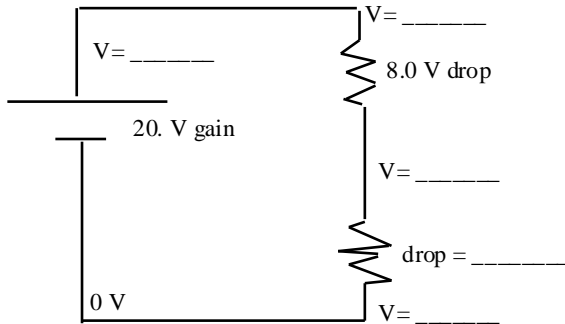
•In terms of the ski hill analogy, KVL simply says that the chairlift takes you to the top of the mountain. From there, as you ski down to the bottom along any path the sum of the drops must equal the height of the mountain.



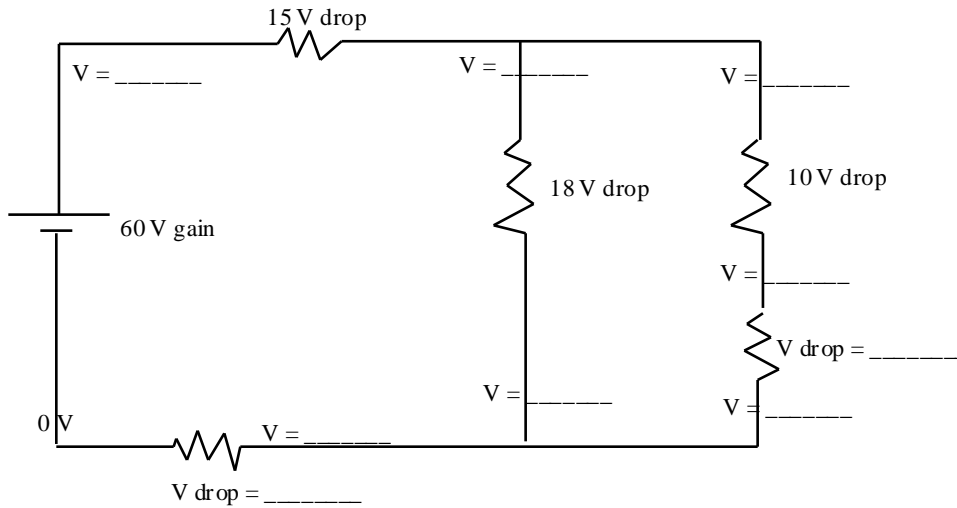
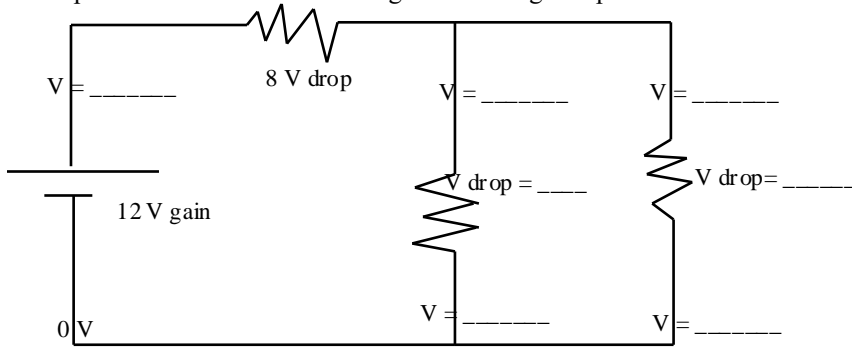
ski hill analogy



•example 4: Find the unknown voltages and voltage drops

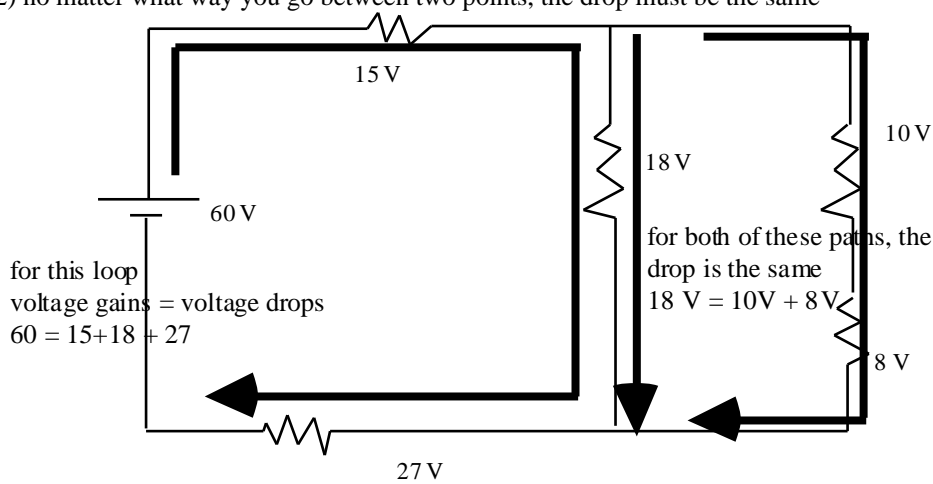


•example 5: find the unknown voltages and voltage drops

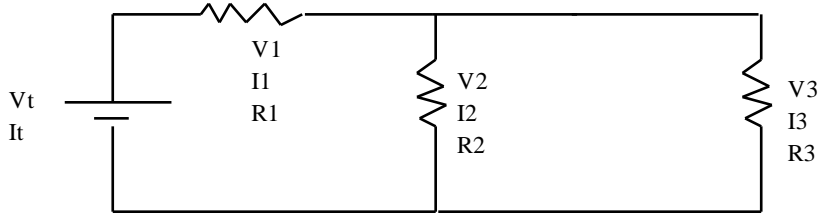


•note that there are really two useful conclusions to draw from KVL:

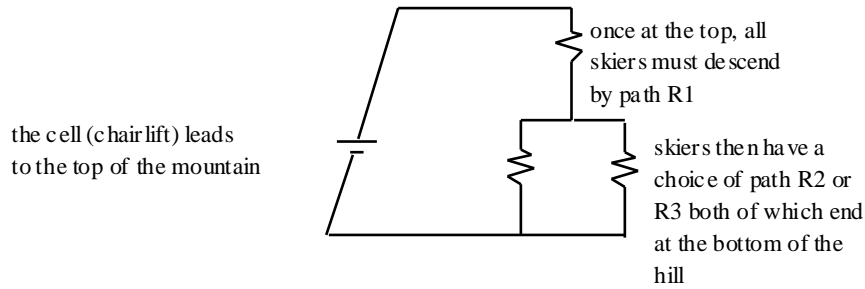
- 1) for any complete loop around the circuit, the voltage drops equal the voltage gains
- 2) no matter what way you go between two points, the drop must be the same



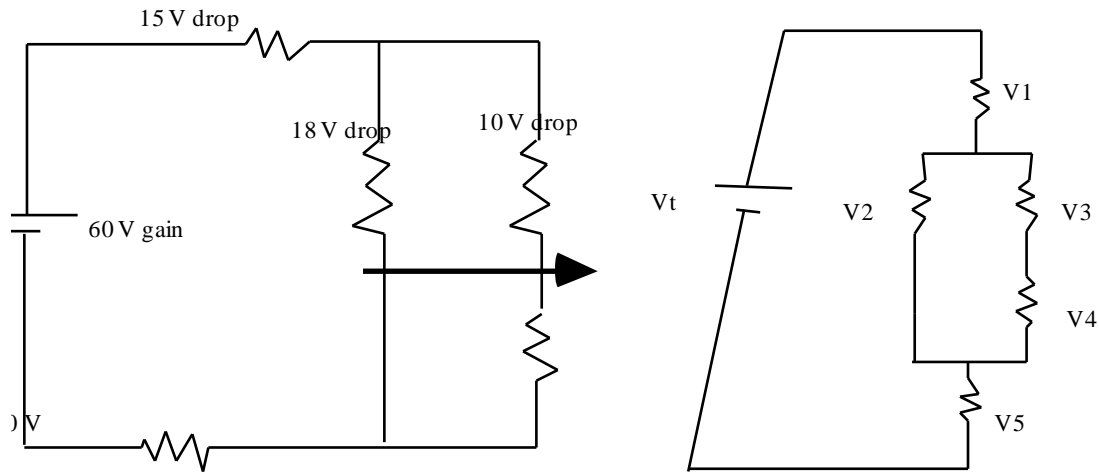
•to see the voltage relationships, it is sometimes helpful (at first) to redraw the circuit so that all resistors are pointing 'downhill'. For instance consider the previous circuit 5a)



this circuit can be re-drawn in the following manner:



and 5b) can be redrawn as follows:



•example 6: write equations relating the voltage drops $V_t, V_1, V_2, V_3, V_4, V_5$ in the above circuits

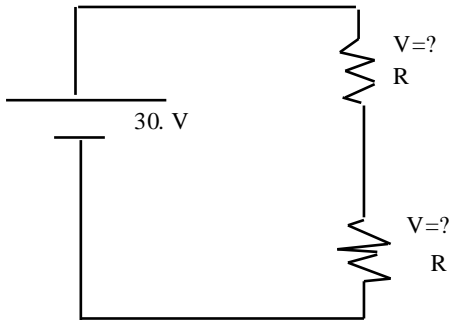
a)

b)

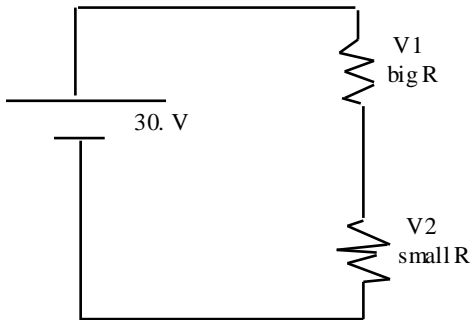
•If there are several resistors along a path, the charges will lose a fraction of their voltage (ie. energy) in each.

•example 7:

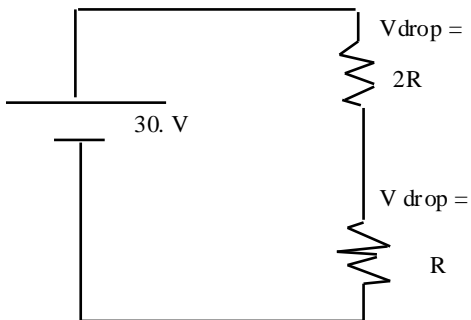
a) if we have two equal resistors along a path, what can be said about the voltage drops? For instance find the voltage drops in the circuit below



b) if the resistances are not the same then what can we say about the division of the voltage? For instance compare the unknown voltage drops V_1 and V_2 below

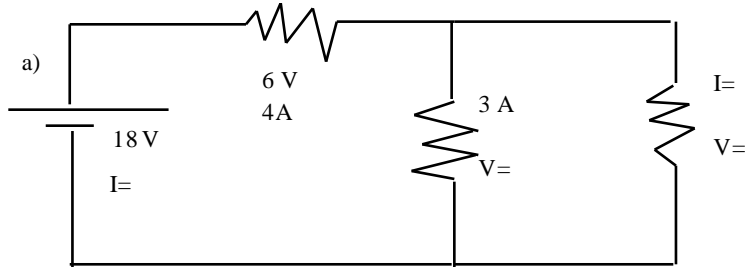


c) if the resistors are simple ratios of each other, we can relate the voltage drops via the same ratios. For instance, find the unknown voltage drops below. (note the first resistor is twice the second)

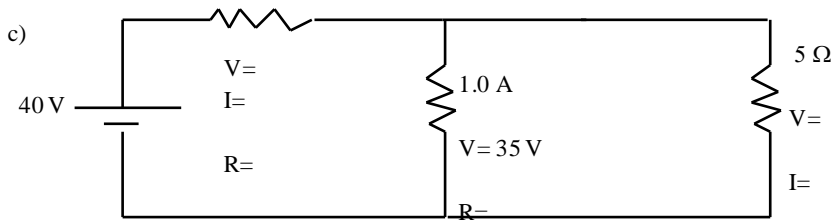
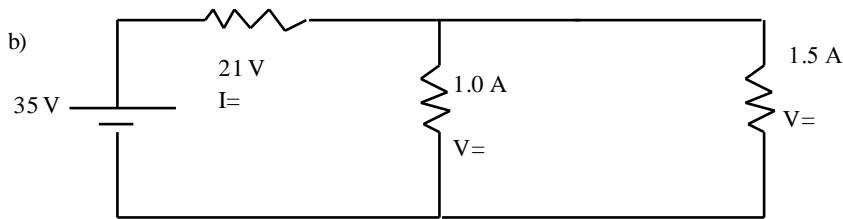


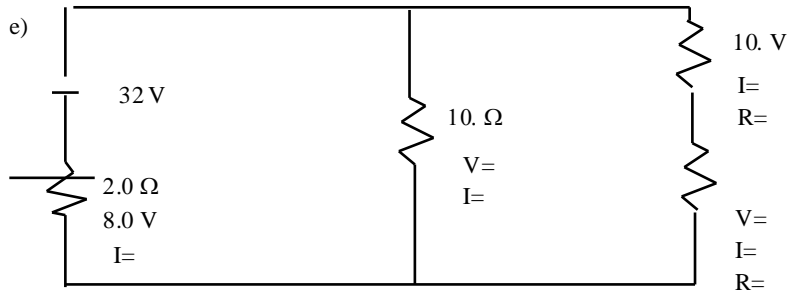
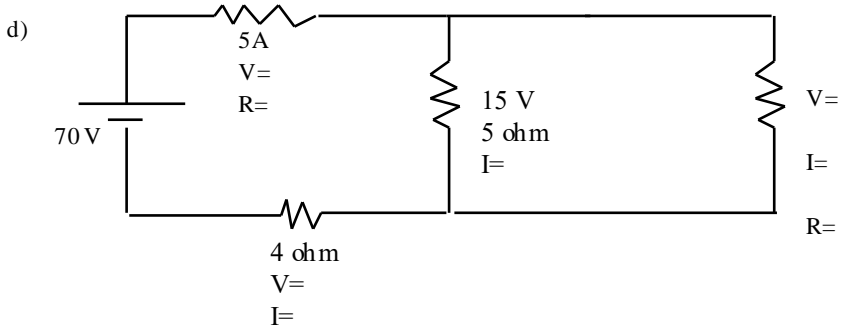
•Although it is helpful at first to label voltages at each point in the circuit, it is desirable at this point to label only the gains and drops. In all of the following examples, the voltages written next to resistors are understood to be the voltage drops. Likewise the voltage numbers written next to cells are the voltage gains.

•example 8: find the unknown voltage drops, currents, resistances, and the power lost in each resistor ($V=IR$)



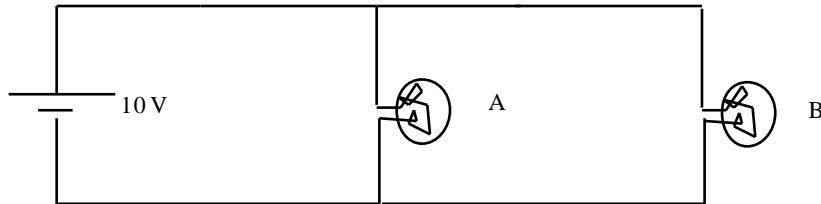
Note: you can build these in the PHET DC Circuit Construction Kit to check your answers!





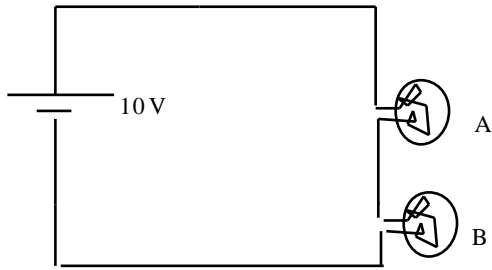
•Lightbulb questions are a good test of student understanding of Kirchhoff's Laws. The brightness of a light bulb is related to the power use (J/s), and therefore depends on the current through a resistor and the voltage drop across the resistor ($P=VI$)

•example 9: the bulbs below are identical(same R)- which bulb is brighter?



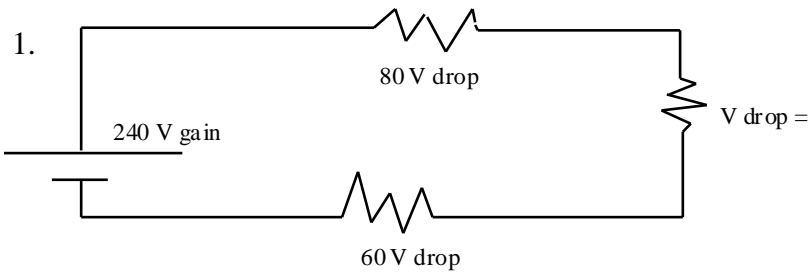
explain your answer using relevant concepts of physics

•example 10: the two bulbs below are identical- which bulb is brighter?

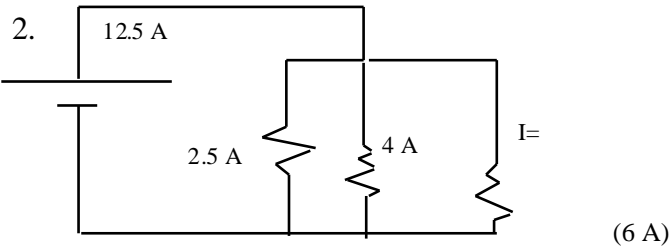


explain your answer using relevant concepts of physics

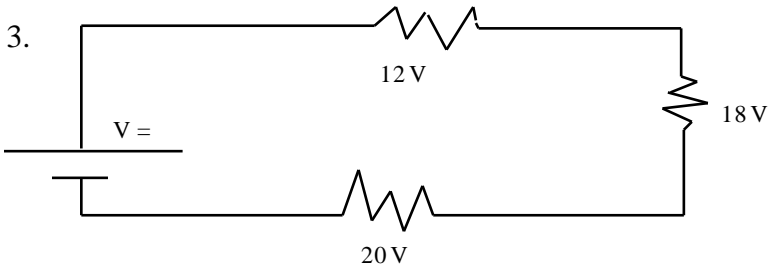
exercises: find unknown voltages and currents



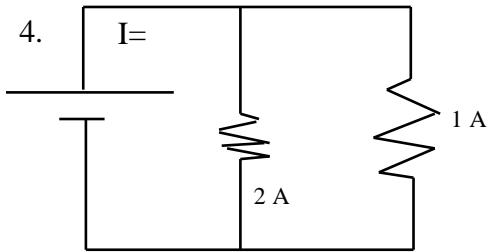
(100V)



(6 A)

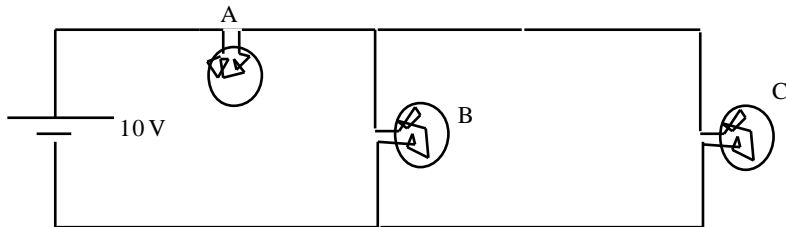


(50V)

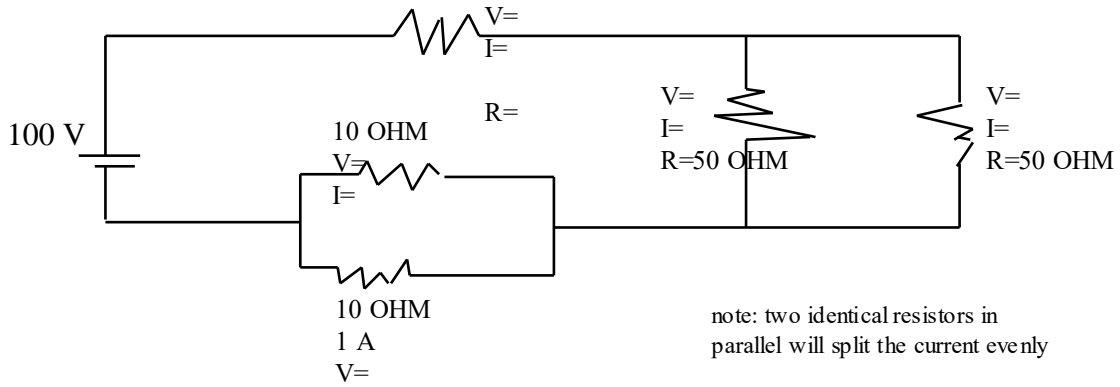


(3A)

5. The three bulbs below are identical- which bulb is brightest? Explain your answer. i) bulb A ii) bulb B iii) bulb C iv) equally bright
 Note that bulb brightness depends on power, and therefore on both current and voltage drop

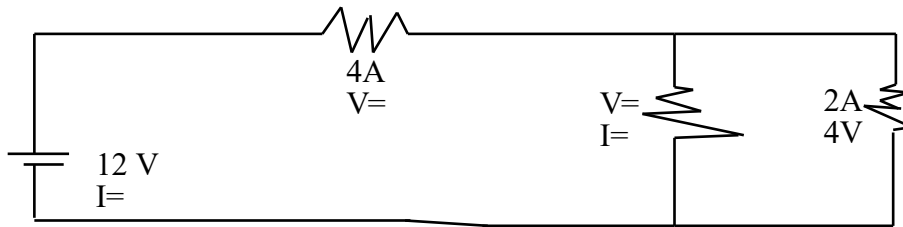


6. Find unknown voltages and currents



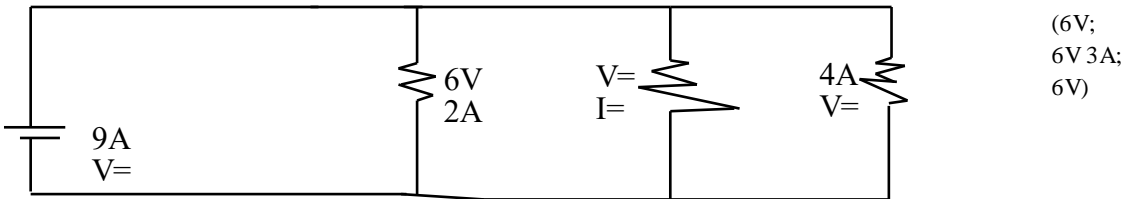
(40V 2A 20 OHM; 50V 1A; 50V 1A; 10V 1A; 10V)

7. Find unknown voltages and currents

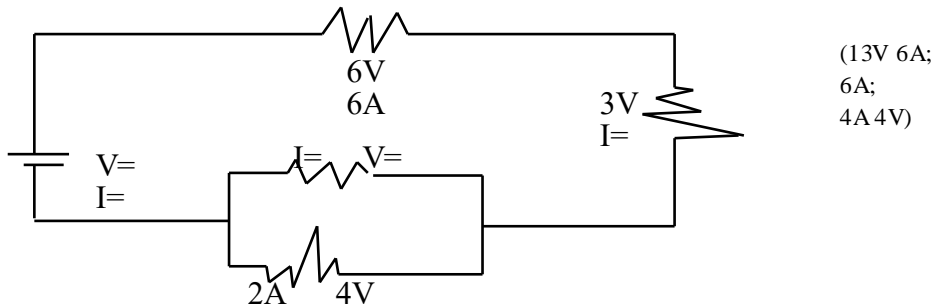


(4A; 8V; 4V 2A)

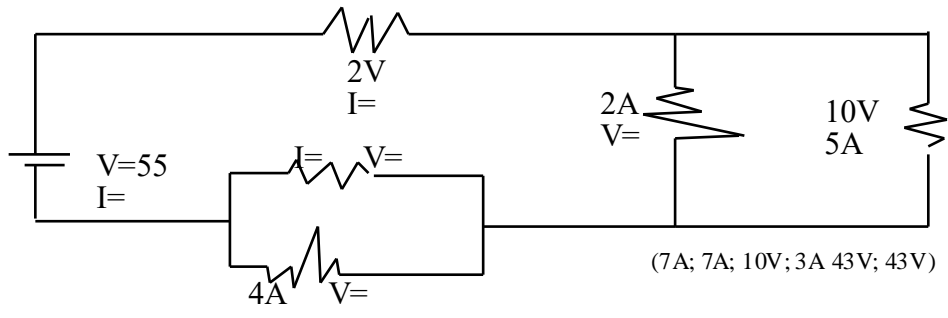
8. Find unknown voltages and currents



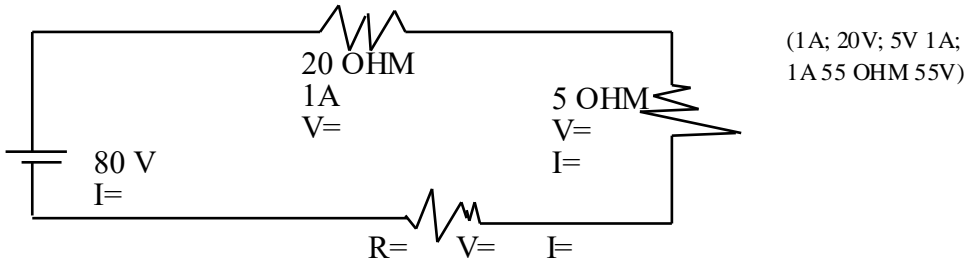
9. Find unknown voltages and currents



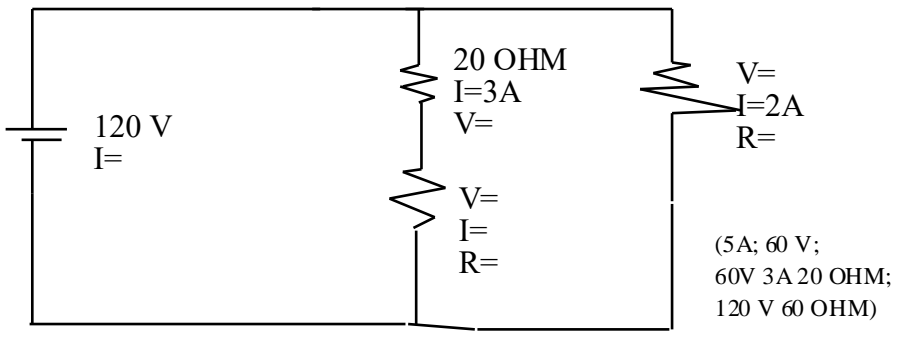
10. Find unknown voltages and currents



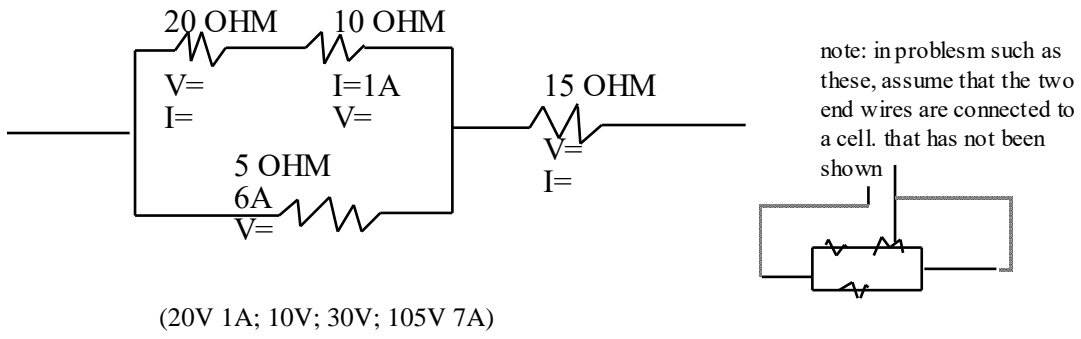
11. Find unknown voltages and currents



12. Find unknown voltages and currents

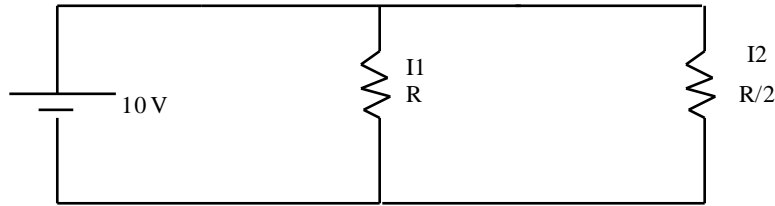


13. Find unknown voltages and currents



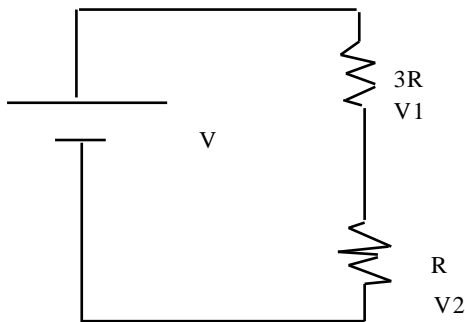
(20V 1A; 10V; 30V; 105V 7A)

14. What can be said about the currents I_1 and I_2 if the resistance of R_2 is half the resistance of R_1 ?



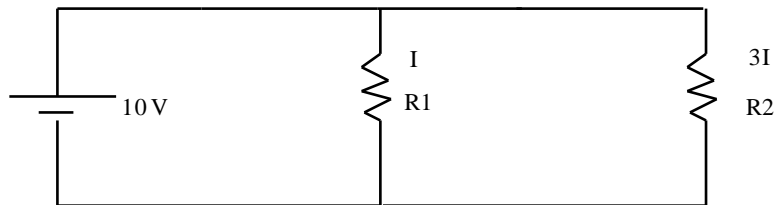
explain your answer using appropriate physics principles

15. What can be said about the voltage drops V_1 and V_2 if the resistance of R_1 is three times the resistance of R_2 ?



explain your answer using appropriate physics principles

16. What can be said about the resistances R_1 and R_2 if the current through R_2 is three times the current through R_1 ?



explain your answer using appropriate physics principles