

January 2008 Solution

Tuesday, June 08, 2010
9:28 PM



Principles of Mathematics 12

Examination Booklet
2007/08 Released Exam
January 2008
Form B

DO NOT OPEN ANY EXAMINATION MATERIALS UNTIL INSTRUCTED TO DO SO.
FOR FURTHER INSTRUCTIONS REFER TO THE RESPONSE BOOKLET.

Contents: 15 pages
44 multiple-choice questions in the Examination Booklet
7 written-response questions in the Response Booklet

Examination: 2 hours
Additional Time Permitted: 60 minutes
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PART A: MULTIPLE CHOICE (non-calculator)
SECTION I

Value: **24** marks

Suggested Time: **35** minutes
Allowable Time: **45** minutes

INSTRUCTIONS: No calculator may be used for this section of the examination. For each question, select the **best** answer and record your choice on the **blue Answer Sheet** provided.

INSTRUCTIONS: No calculator may be used for this section of the examination. For each question, select the best answer and record your choice on the blue Answer Sheet provided. Using an HB pencil, completely fill in the bubble that has the letter corresponding to your answer.

You have Examination Booklet Form B. In the box above #1 on your Answer Sheet, fill in the bubble as follows.

Exam Booklet Form/ Cahier d'examen	A	B	C	D	E	F	G	H
	<input type="radio"/>	<input checked="" type="radio"/>	<input type="radio"/>					

1. Determine the amplitude of the function $y = -4 \cos(x - 2)$.

- A. -4
- B. -2
- C. 2
- D. 4

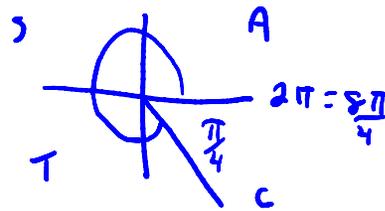
2. The terminal arm of angle θ in standard position intersects the unit circle at the point (m, n) . Which expression represents $\tan \theta$?

- A. $\frac{n}{m}$
- B. $\frac{m}{n}$
- C. $\frac{1}{m}$
- D. $\frac{1}{n}$

$$\frac{n}{m} = \frac{y}{x}$$

3. Determine the exact value of $\csc \frac{7\pi}{4}$.

- A. $-\sqrt{2}$
- B. $\sqrt{2}$
- C. $-\frac{1}{\sqrt{2}}$
- D. $\frac{1}{\sqrt{2}}$



$$\csc = \frac{h}{o} = \frac{-\sqrt{2}}{1}$$

- C. $-\frac{1}{\sqrt{2}}$
- ~~D. $\frac{1}{\sqrt{2}}$~~

$\frac{\sqrt{2}}{1}$

$\frac{0}{1} = \frac{0}{1}$

$\swarrow 60^\circ \times 2$

4. In a circle, an arc of length 30 cm contains a central angle of 120° . Determine the radius of this circle.

- A. $\frac{36}{\pi}$ cm
- B. $\frac{45}{\pi}$ cm**
- C. 20π cm
- D. 45π cm

$a = r\theta$

$120^\circ \times \frac{\pi}{180^\circ}$

$r = \frac{a}{\theta} = \frac{30}{\frac{2\pi}{3}} = 30 \times \frac{3}{2\pi} = \frac{90}{2\pi} = \frac{45}{\pi}$

5. Determine an equivalent expression for $\sin 3x \cos x + \cos 3x \sin x$.

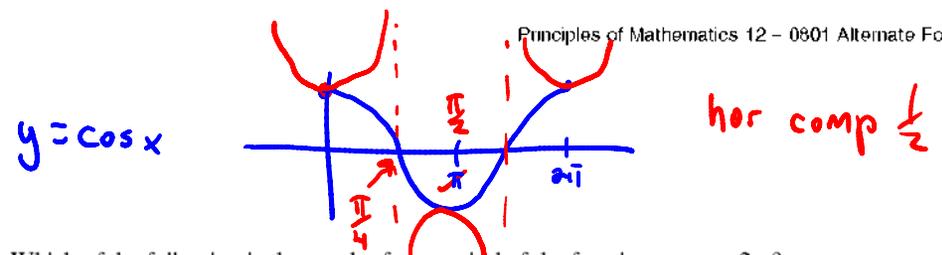
- ~~A. $4 \sin x$~~
- B. $2 \sin x \cos x$
- C. $4 \sin x \cos x$
- D. $2 \sin 2x \cos 2x$**

$\sin \alpha \cos \beta + \cos \alpha \sin \beta$
 $= \sin(\alpha + \beta) = \sin(3x + x)$
 $= \sin 4x \quad A = 2x$

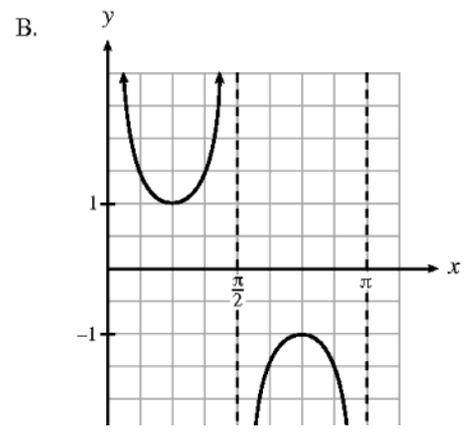
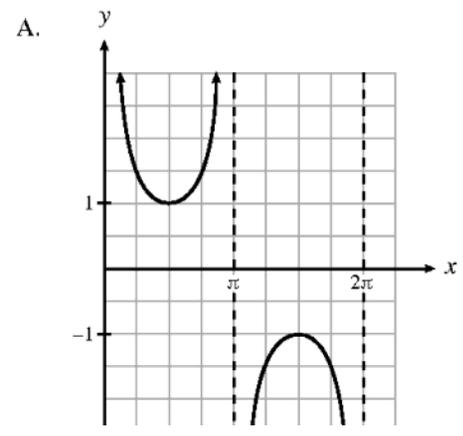
$= \sin(2A) = 2 \sin A \cos A = 2 \sin 2x \cos 2x$

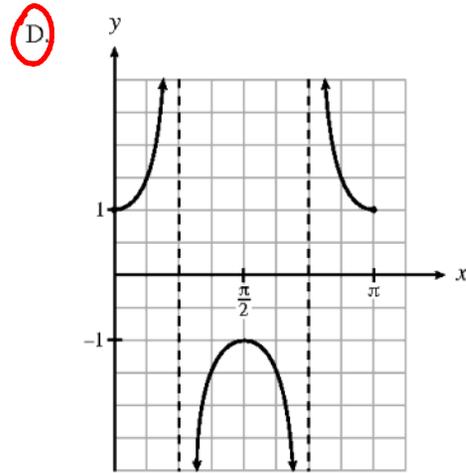
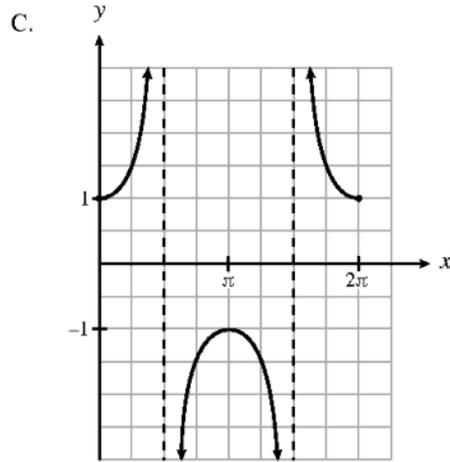
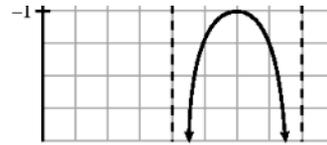
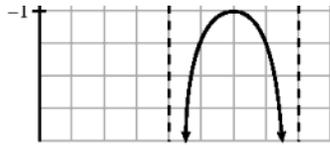
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Principles of Mathematics 12 – 0601 Alternate Form B

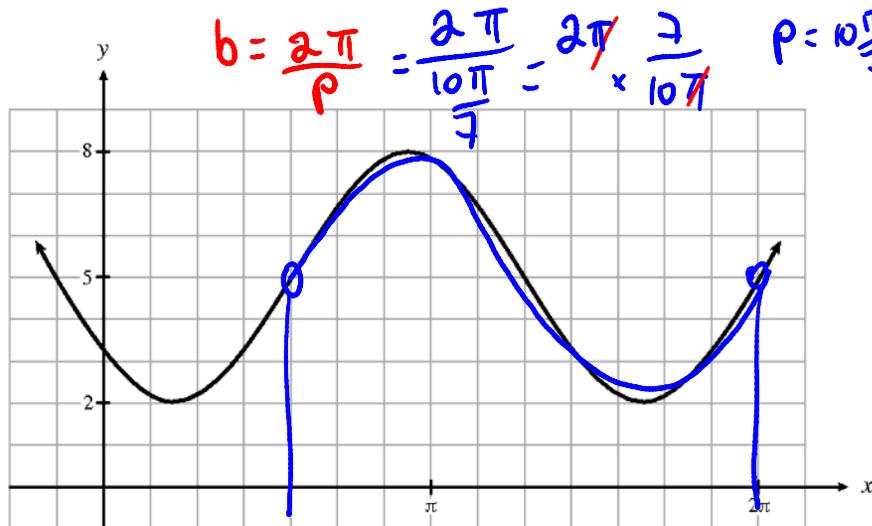


6. Which of the following is the graph of one period of the function $y = \sec 2x$?





7. Given the graph of the function $y = a \sin b(x - c) + d$ below, determine the value of b .





- A. $\frac{10\pi}{7}$
- B. $\frac{7}{5}$**
- C. $\frac{5\pi}{7}$
- D. $\frac{14}{5}$

8. Solve: $4\cos^2 x = 3, 0 \leq x < 2\pi$

~~A. $\frac{\pi}{6}, \frac{11\pi}{6}$~~

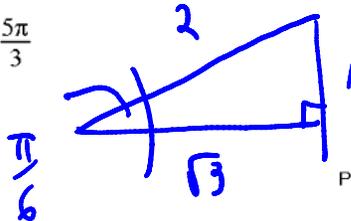
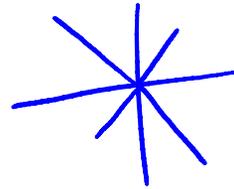
B. $\frac{\pi}{6}, \frac{5\pi}{6}, \frac{7\pi}{6}, \frac{11\pi}{6}$

~~C. $\frac{\pi}{3}, \frac{5\pi}{3}$~~

D. $\frac{\pi}{3}, \frac{2\pi}{3}, \frac{4\pi}{3}, \frac{5\pi}{3}$

$\cos^2 x = \frac{3}{4}$

$\cos x = \pm \frac{\sqrt{3}}{2}$

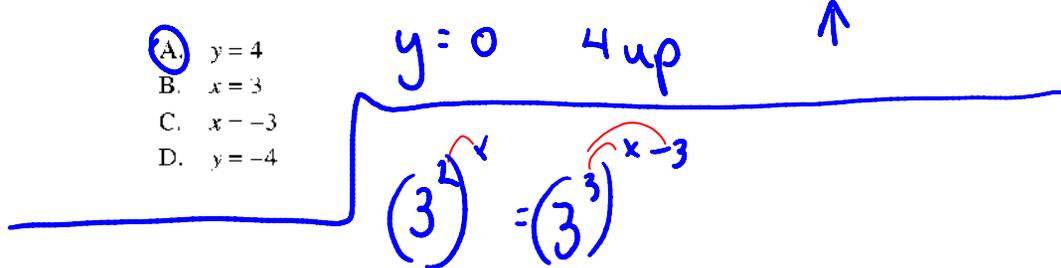


Page 4

Principles of Mathematics 12 – 0801 Alternate Form B

9. Determine an equation for the asymptote of the graph of $y = 2^{x+3} + 4$.

- A. $y = 4$**
- B. $x = 3$
- C. $x = -3$
- D. $y = -4$



10. Solve: $9^x = 27^{x-3}$

- A. -9
- B. 3
- C. $\frac{9}{2}$
- D. 9**

$3^{2x} = 3^{3x-9}$

$2x = 3x - 9$
 $-3x -3x$

$-x = -9$
 $x = 9$

11. Solve: $\log_5(3x) - \log_5(x-3) = 2$

11. Solve: $\log_5(3x) - \log_5(x-3) = 2$

- A. -6
- B. $\frac{1}{2}$
- C. $\frac{75}{22}$
- D. 11

$$\log_5\left(\frac{3x}{x-3}\right) = 2 \rightarrow \frac{3x}{x-3} = 5^2$$

$$\frac{3x}{x-3} = \frac{25}{1} \rightarrow 3x = 25x - 75$$
$$-22x = -75 \quad x = \frac{75}{22}$$

12. Evaluate: $\log_5 \sqrt{5^3}$

- A. $\frac{1}{6}$
- B. $\frac{2}{3}$
- C. $\frac{3}{2}$
- D. 6

$$\log_5 (5^3)^{\frac{1}{2}} = \log_5 5^{\frac{3}{2}}$$
$$= \frac{3}{2} \log_5 5$$

13. If $\log_2 5 = x$ and $\log_2 3 = y$, determine an expression for $\log_2 \left(\frac{15}{2}\right)$, in terms of x and y . ← 3×5

- A. xy
 - B. $x+y$
 - C. $xy-1$
 - D. $x+y-1$
- $$= \frac{\log 5}{2} + \frac{\log 3}{2} - \frac{\log 2}{2}$$

$$= x + y - 1$$

14. Solve: $\log_2(\log_4(x+6)) = 1$

- A. 2
 - B. 3
 - C. 2, 3
 - D. -2, 3
- $2 = \log_2(\log_4(x+6))$
 $x^2 = x + 6$
- $a^2 - x - 6 = 0$
 $(x-3)(x+2) = 0$
 $x = 3, -2$

15. Evaluate: $\frac{6!}{3!2!}$

- A. 1
 - B. 6
 - C. 30
 - D. 60
- $$= \frac{6 \times 5 \times 4 \times 3 \times 2 \times 1}{3 \times 2 \times 1 \times 2}$$

16. If $f(x) = (x+4)(x-2)$, determine the zeros of the function $y = f(2x)$.

- A. -8, 4
 - B. -4, 2
 - C. -2, 1
 - D. -1, 2
- $2x+4 = 0 \rightarrow x = -2$
 $2x-2 = 0 \rightarrow x = 1$
- hor comp $\frac{1}{2}$

This is the end of Part A, Section I.

You may proceed to the rest of the examination *without* the use of a calculator until directed by the supervisor to access your calculator. At the end of 45 minutes, you will not be able to go back to Part A, Section I; therefore, ensure you have checked this section.

PART A: MULTIPLE CHOICE
SECTION II

Value: 42 marks

Suggested Time: 55 minutes

INSTRUCTIONS: For each question, select the **best** answer and record your choice on the **white Answer Sheet** provided. Using an HB pencil, completely fill in the bubble that has the letter corresponding to your answer.

17. Which equation represents the graph of $y = f(x)$ after it is expanded vertically by a factor of 5?

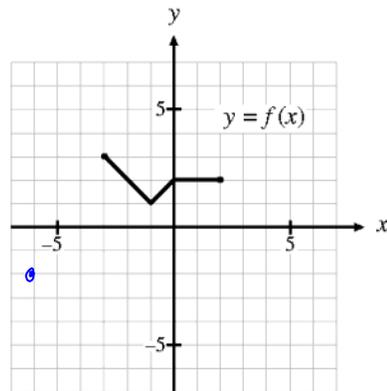
A. $y = \frac{1}{5}f(x)$

B. $y = 5f(x)$

C. $y = f\left(\frac{x}{5}\right)$

D. $y = f(5x)$

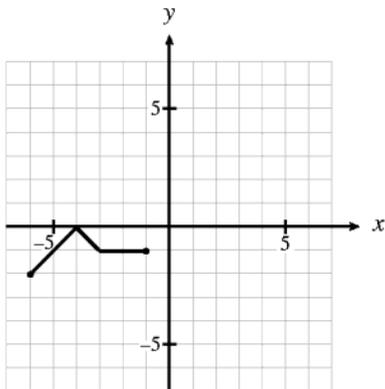
18. The graph of $y = f(x)$ is shown below.



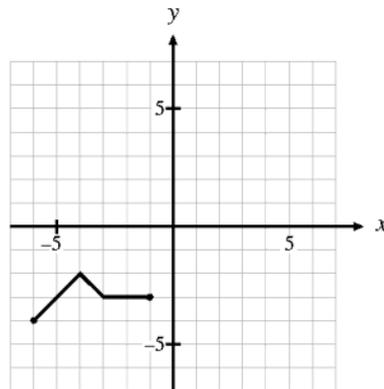
vert refl
3L
1U

Which graph represents the graph of $y = -f(x + 3) + 1$?

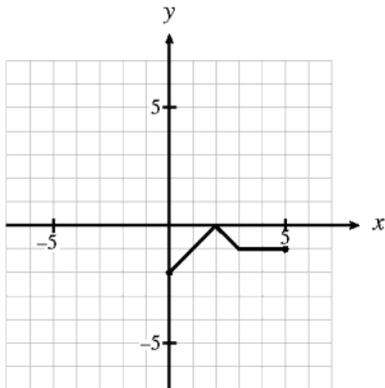
A.



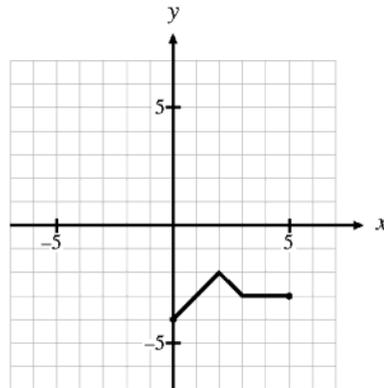
B.



C.



D.



19. Determine the inverse of the function $f(x) = x^3 - 2$.

A. $f^{-1}(x) = \sqrt[3]{x+2}$

B. $f^{-1}(x) = \sqrt[3]{x} + 2$

C. $f^{-1}(x) = \sqrt[3]{x} - 2$

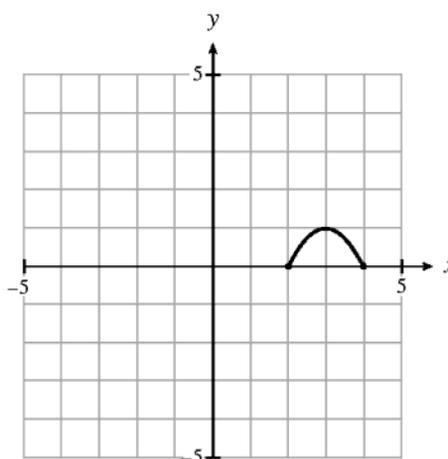
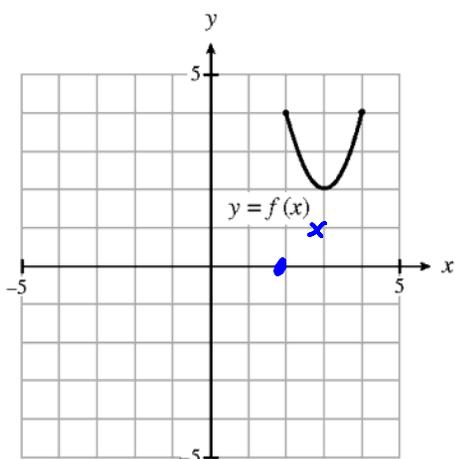
D. $f^{-1}(x) = \sqrt[3]{x-2}$

$x = y^3 - 2$

$y^3 = x + 2$

$y = \sqrt[3]{x+2}$

20. The graph of $y = f(x)$ is shown below on the left. Determine an equation of the function graphed on the right.



A. $y = -\frac{1}{2}f(x)$

B. $y = -\frac{1}{2}f(x) + 2$

C. $y = -\frac{1}{2}f(x) + 3$

D. $y = -\frac{1}{2}f(x) + 4$

vert refl
vert comp $\frac{1}{2}$

~~$0 = -\frac{1}{3}x - 12$~~

21. If the point $(6, -12)$ is on the graph of $y = f(x)$, which point must be on the graph

of $y = f\left(-\frac{1}{3}x + 6\right)$? $= f\left(-\frac{1}{3}(x - 18)\right)$

$$0 = 10x_1 - 12$$

21. If the point $(6, -12)$ is on the graph of $y = f(x)$, which point must be on the graph

$$\text{of } y = f\left(-\frac{1}{3}x + 6\right)? = f\left(-\frac{1}{3}(x - 18)\right)$$

- A. $(36, 12)$
- B. $(-24, -12)$
- C. $(0, -12)$
- D. $(16, -12)$

hor. exp 3
 hori refl
 18 R

22. Solve: $5 \sin^2 x = \cos x$, $0 \leq x < 2\pi$

- A. 0.43, 1.78
- B. 0.44, 5.84
- C. 0.82, 1.73
- D. 2.87, 3.58

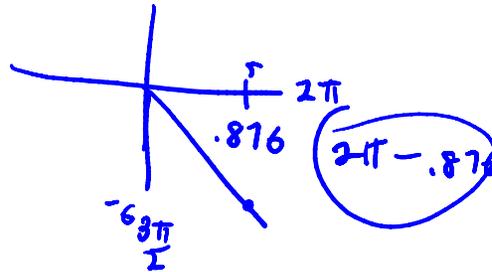
23. The point $(5, -6)$ is on the terminal arm of standard position angle θ . Determine the smallest positive measure of θ in radians.

- A. 0.69
- B. 0.88
- C. 5.41
- D. 5.59

$$x = 5 \quad y = -6$$

$$\tan \theta = -\frac{6}{5}$$

$$R(\theta) = \tan^{-1}\left(\frac{6}{5}\right)$$

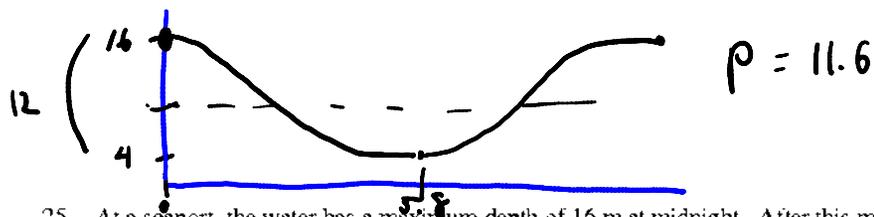


24. Determine the restriction(s) for the expression $\frac{\sec x}{2 \sin x + 1}$.

- A. $\sin x \neq -\frac{1}{2}$
- B. $\sin x \neq 0$, $\sin x \neq \frac{1}{2}$
- C. $\cos x \neq 0$, $\sin x \neq -\frac{1}{2}$
- D. $\cos x \neq 0$, $\sin x \neq 0$, $\sin x \neq -\frac{1}{2}$

$$\sin x \neq -\frac{1}{2} \quad \frac{1}{\cos x}$$

$$\cos x \neq 0$$



25. At a seaport, the water has a maximum depth of 16 m at midnight. After this maximum depth, the first minimum depth of 4 m occurs 5.8 h later. Assume that the relation between the depth in metres and the time in hours is a sinusoidal function. How many hours after midnight will the water reach a depth of 8 m for the first time?

- A. 1.76 h
 B. 2.27 h
 C. 3.53 h
 D. 3.67 h

$$h = 6 \cos \frac{2\pi}{11.6}(t) + 10$$

26. Which of the following is a geometric sequence?

- A. 1, 4, 9, ...
 B. 2, 4, 10, ...
 C. 12, 7, 2, ...
 D. 18, 12, 8, ...

27. Determine the number of terms in the geometric sequence 3, 6, 12, ..., 49152.

- A. 13
 B. 14
 C. 15
 D. 16

$a = ?$
 $a = 3$
 $r = 2$

$$t_n = 49152$$

$$t_n = ar^{n-1}$$

$$49152 = 3(2)^{n-1}$$

$$16384 = 2^{n-1}$$

28. Determine the sum of the infinite geometric series $16 - 12 + 9 - \dots$

- A. $\frac{48}{7}$
 B. $\frac{64}{7}$
 C. 64
 D. no finite sum

$$S = \frac{a}{1-r}$$

$$= \frac{16}{1 - .75} =$$

$$a = 16$$

$$r = \frac{-12}{16} = -.75$$

33. A population grows continuously according to the formula $P = P_0 e^{kt}$, where P is the final population at the end of t years, P_0 is the initial population, and k is the annual growth rate. What will the population be at the end of 10 years if the initial population is 5000 and the annual growth rate is 3%?

A. 6720
 B. 6749
 C. 51523
 D. 100428

$P = 5000 \times e^{(.03)(10)}$
 $P =$

34. In 1872, Washington State experienced an earthquake of magnitude 6.8 on the Richter scale. Determine the magnitude on the Richter scale of an earthquake that is half as intense as the Washington State earthquake.

A. 3.4
 B. 6.0
 C. 6.5
 D. 7.1

10 bigger - smaller = times
 $6.8 - x = 2$
 $6.8 - x = \log 2$
 $x = 6.8 - \log 2$
 $(6.8 - x) \log 10 = \log 2$
 $6.8 - x = \log 2$
 $x = 6.8 - \log 2$

35. A coach needs to choose an 8-member volleyball team from 10 males and 12 females. If there must be at least 3 of each gender on the team, how many different teams are possible?

A. 103950
 B. 150480
 C. 254430
 D. 319770

3M or 4M or 5M
 $(10C_3)(12C_5) + (10C_4)(12C_4) + (10C_5)(12C_3) = 103950 + 103950 + 55440 = 254430$

36. How many even 4-digit whole numbers are there? For example, 1220 is acceptable but 0678 is not.

A. 3600
 B. 3645
 C. 4500
 D. 5000

Repetitions allowed!

$\frac{9}{\emptyset} \quad \frac{10}{\quad} \quad \frac{10}{\quad} \quad \frac{5}{0,2,4,6,8}$

37. An Olympic final race has 7 competitors. In how many possible ways could the gold, silver and bronze medals be awarded?

A. 21
 B. 35
 C. 210
 D. 5040

$7P_3 = \frac{7!}{4!}$ 7 6 5

38. If $64a^6$ is the 1st term in the expansion of $(2a - b)^n$, determine the coefficient of the 4th term.

A. -160
 B. -20
 C. 20
 D. 160

$t_{k+1} = nC_k a^{n-k} b^k$ $k=3$
 $n=6$
 $a=2a$
 $b=-b$

$t_4 = 6C_3 (2a)^3 (-b)^3$
 $= -160 a^3 b^3$

39. Each day a student chooses 1 out of 3 beverages in the school cafeteria. Over 10 days he chooses apple juice 3 times, orange juice 3 times and lemonade 4 times. In how many different orders can this occur?

A. 360
 B. 4 200
 C. 3 628 800
 D. 87 091 200

AAA OOO LLLL

$\frac{10!}{3!3!4!} = \frac{10!}{6 \times 6 \times 24} = 4200$

40. It is known that 22% of households have dogs as pets and 13% have cats. If 3% have both, what percent have neither?

A. 32%
 B. 62%
 C. 65%
 D. 68%

41. A card is drawn from a standard deck of 52 cards and then replaced. In 20 draws of this type, what is the probability that a red card will be drawn exactly 12 times?

A. 0.0008
 B. 0.1198
 C. 0.1201
 D. 0.8684

$p = \frac{1}{2}$

${}_{20}C_{12} (.5)^{12} (.5)^8$

or binompdf(20, .5, 12) =

42. Two fair 6-sided dice are rolled and the face values showing are added. What is the probability that the sum of the face values is at least 9?

A. $\frac{5}{6}$
 B. $\frac{5}{18}$ (st)
 C. $\frac{1}{6}$
 D. $\frac{1}{9}$

2nd: 1 2 3 4 5 6

1 1, 1 1, 2
 2
 3
 4
 5
 6

6, 3
 5, 4
 6, 4
 5, 5
 6, 5
 4, 6
 5, 6
 6, 6

$\frac{10}{36}$

43. A rock concert is attended by 1300 students of whom 760 are girls and 540 are boys. If 4 of these students are randomly chosen to receive backstage passes, what is the probability that at most 3 of the chosen students will be boys?

A. 0.0296
 B. 0.1674
 C. 0.8836
 D. 0.9704

$1 - P(4 \text{ Boys})$
 $1 - \frac{540C_4 \times 760C_0}{1300C_4} =$

0, 1, 2, 3
 $4 \left[\begin{array}{c|c} 540 & 760 \end{array} \right]_0$
 B G

44. Ten discs numbered 0 to 9 are placed in a box. Two discs are randomly removed from the box without replacement. Determine the probability that one disc will have the digit 5 written on it and the other disc will have the digit 6 written on it.

A. $\frac{1}{45}$
 B. $\frac{1}{50}$
 C. $\frac{1}{90}$
 D. $\frac{1}{100}$

$\frac{2C_2 \times 8C_0}{10C_2} = \frac{1}{45}$

2 ← 2 | 8 → 0
 W L
 5, 6

You have Examination Booklet Form B. In the box above #1 on your Answer Sheet, ensure you filled in the bubble as follows.

Exam Booklet Form/ Cahier d'examen	A	B	C	D	E	F	G	H
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This is the end of the multiple-choice section.
 Answer the remaining questions directly in the Response Booklet.

A SUMMARY OF BASIC IDENTITIES AND FORMULAE

Pythagorean Identities:

$$\sin^2 \theta + \cos^2 \theta = 1$$

$$1 + \tan^2 \theta = \sec^2 \theta$$

$$1 + \cot^2 \theta = \csc^2 \theta$$

Reciprocal and Quotient Identities:

$$\sec \theta = \frac{1}{\cos \theta}$$

$$\csc \theta = \frac{1}{\sin \theta}$$

$$\cot \theta = \frac{1}{\tan \theta}$$

$$\tan \theta = \frac{\sin \theta}{\cos \theta}$$

$$\cot \theta = \frac{\cos \theta}{\sin \theta}$$

Addition Identities:

$$\cos(\alpha + \beta) = \cos \alpha \cos \beta - \sin \alpha \sin \beta$$

$$\sin(\alpha + \beta) = \sin \alpha \cos \beta + \cos \alpha \sin \beta$$

$$\cos(\alpha - \beta) = \cos \alpha \cos \beta + \sin \alpha \sin \beta$$

$$\sin(\alpha - \beta) = \sin \alpha \cos \beta - \cos \alpha \sin \beta$$

Double-Angle Identities:

$$\cos 2\theta = \cos^2 \theta - \sin^2 \theta$$

$$\sin 2\theta = 2 \sin \theta \cos \theta$$

$$= 2 \cos^2 \theta - 1$$

$$= 1 - 2 \sin^2 \theta$$

Formulae:

$$t_n = ar^{n-1} \quad S_n = \frac{a(1-r^n)}{1-r} \quad S_n = \frac{a-r^n}{1-r} \quad S = \frac{a}{1-r} \quad x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

Combinatorics and Probability:

$${}_n P_r = \frac{n!}{(n-r)!} \quad {}_n C_r = \binom{n}{r} = \frac{n!}{r!(n-r)!} \quad t_{k+1} = {}_n C_k a^{n-k} b^k$$

$$P(\bar{A}) = 1 - P(A)$$

$$P(A \text{ or } B) = P(A) + P(B) - P(A \text{ and } B)$$

$$P(A|B) = \frac{P(A \text{ and } B)}{P(B)}$$

$$P(A \text{ and } B) = P(A) \times P(B|A)$$

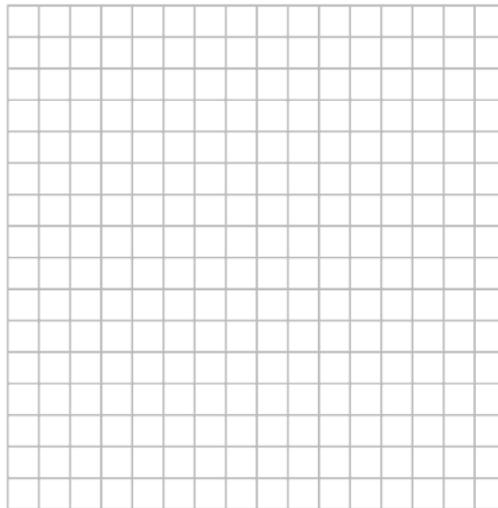
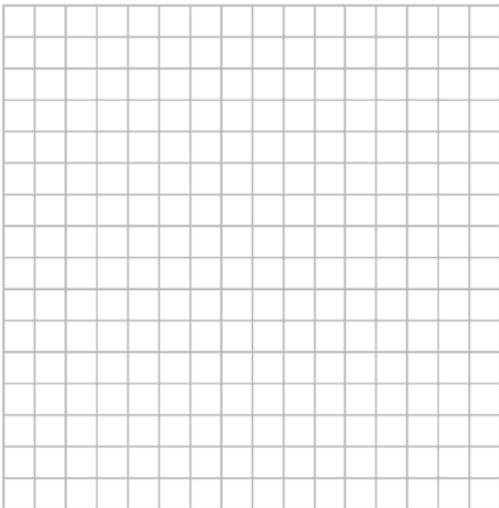
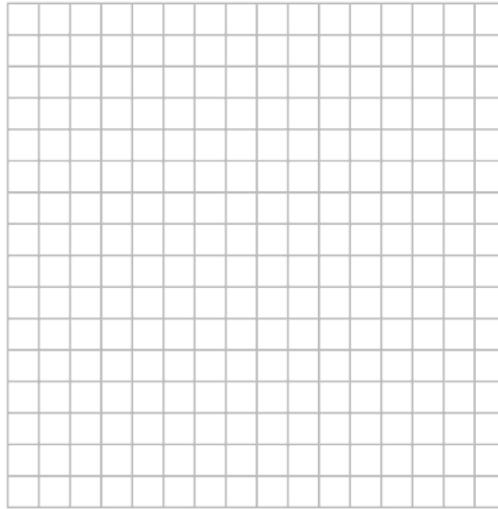
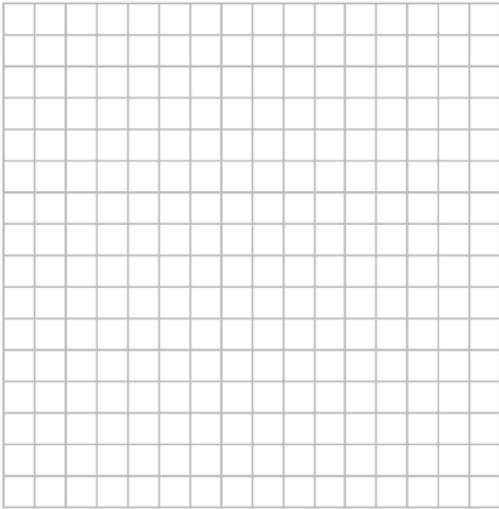
$$P(x) = {}_n C_x p^x q^{n-x}$$

$$q = 1 - p$$

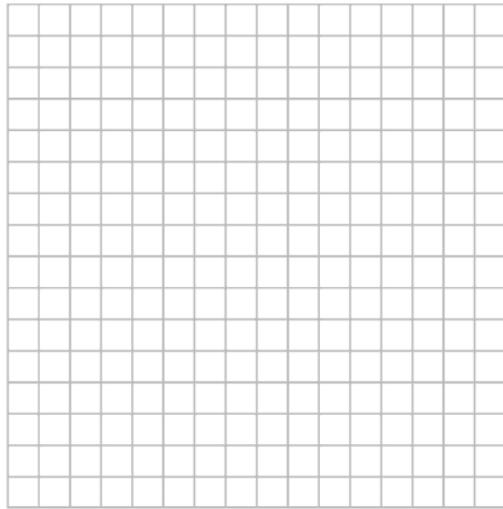
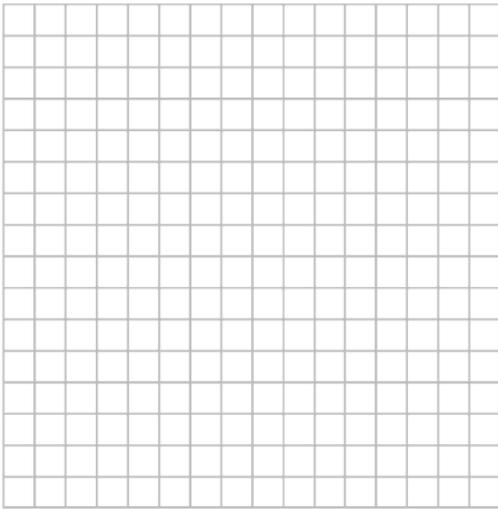
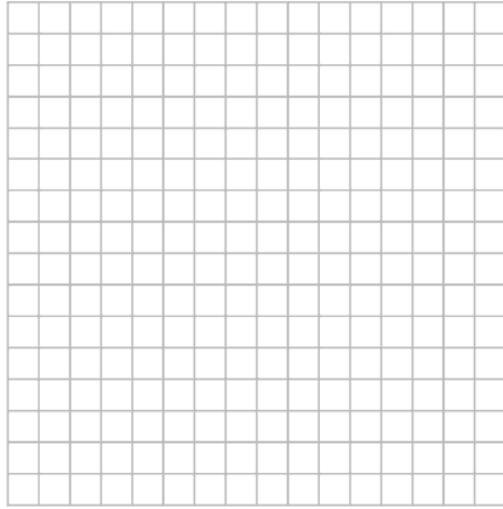
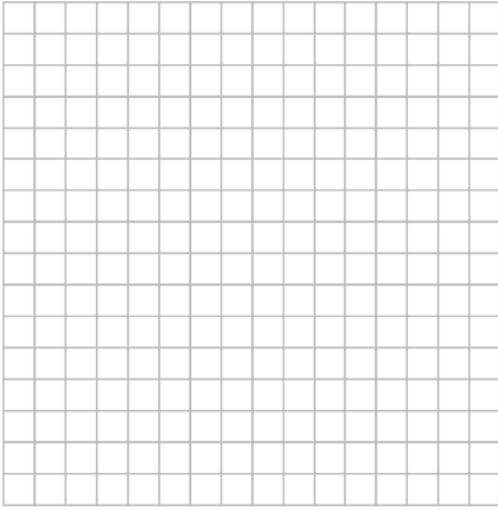
Note: Graphing calculators will contain many of these formulae as pre-programmed functions.

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ROUGH WORK FOR GRAPHING
(No marks will be given for work done on this page.)



ROUGH WORK FOR GRAPHING
(No marks will be given for work done on this page.)



ROUGH WORK FOR MULTIPLE CHOICE

Data Page 8

Principles of Mathematics 12

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Place Personal Education Number (PEN) here.

←→

Course Code = MA 12
JANUARY 2008 Alternate

Exam Booklet Form/ A B C D E F G H
Cahier d'examen

Student Instructions

1. Place your Personal Education Number (PEN) label at the top of this Booklet. On each of your Answer Sheets (one blue bubble sheet and one white bubble sheet) fill in the bubble (Form A, B, C, D, E, F, G or H) that corresponds to the letter on your Examination Booklet.
2. Use a pencil to fill in bubbles when answering questions on your Answer Sheet.
3. When answering questions in Section I (45 minutes):
 - use the blue answer sheet.
 - calculators are not permitted.
 - you may proceed to other questions that do not require the use of a calculator if you finish this section early. Note: no calculator will be allowed for the first 45 minutes of the examination.
 - you will NOT be able to return to this section after the time limit.
4. When using a calculator:
 - round final answers with decimals to at least two decimal places unless otherwise indicated in the question.
5. Read the Examination Rules on the back of this Booklet.

MINISTRY USE ONLY

Question 1							
0	1	2	3			(.5)	NR
<input type="checkbox"/>	<input type="checkbox"/>	<input type="checkbox"/>	<input type="checkbox"/>			<input type="checkbox"/>	<input type="checkbox"/>
Question 2							
0	1	2				(.5)	NR
<input type="checkbox"/>	<input type="checkbox"/>	<input type="checkbox"/>				<input type="checkbox"/>	<input type="checkbox"/>
Question 3							
0	1	2	3	4	5	(.5)	NR
<input type="checkbox"/>							
Question 4							
0	1	2	3	4	5	(.5)	NR
<input type="checkbox"/>							
Question 5							
0	1	2	3			(.5)	NR
<input type="checkbox"/>	<input type="checkbox"/>	<input type="checkbox"/>	<input type="checkbox"/>			<input type="checkbox"/>	<input type="checkbox"/>
Question 6							
0	1					(.5)	NR
<input type="checkbox"/>	<input type="checkbox"/>					<input type="checkbox"/>	<input type="checkbox"/>
Question 7							
0	1	2	3	4	5	(.5)	NR
<input type="checkbox"/>							

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Course Code = MA 12

**Principles of
Mathematics 12
2007/08 Released Exam
JANUARY 2008 Alternate
Response Booklet**



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PART B: WRITTEN RESPONSE

Value: 24 marks

Suggested Time: 30 minutes

INSTRUCTIONS: Answer the following questions in the space provided in the **Response Booklet**.

Rough-work space has been incorporated into the space allowed for answering each question. You may not need all the space provided to answer each question.

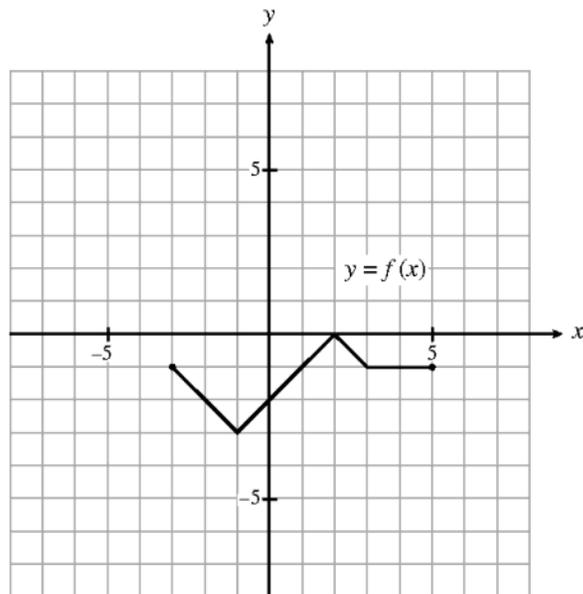
If, in a justification, you refer to information generated by a graphing calculator, this information must be presented clearly in the response. If the statistical features of the calculator are used, it is important to show the function with the substitution of the relevant numbers. For example: in part of the solution it is acceptable to show `binomcdf (20, 0.5, 10)` or the equivalent syntax for the calculator used.

When using the calculator, you should provide a decimal answer that is **accurate to at least two decimal places** (unless otherwise indicated). Such rounding should occur **only** in the final step of the solution.

Full marks will NOT be given for a final answer only.

Use the following graph to answer questions 1 and 2.

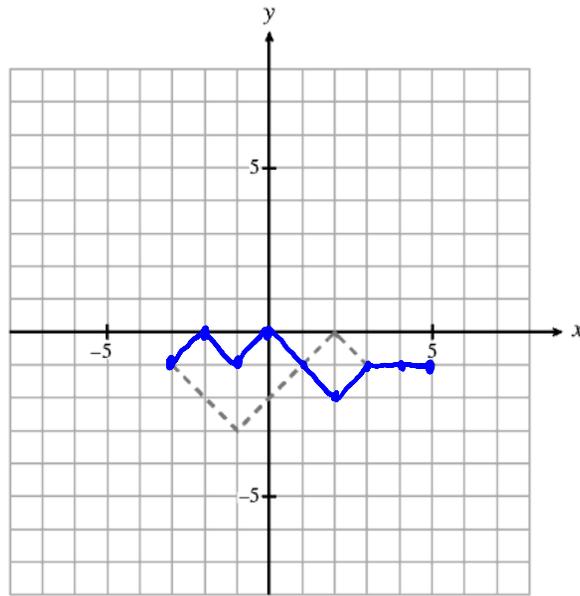
The graph of $y = f(x)$ is shown below.



1. On the grid provided, sketch the graph of $y = -|f(x) + 2|$.

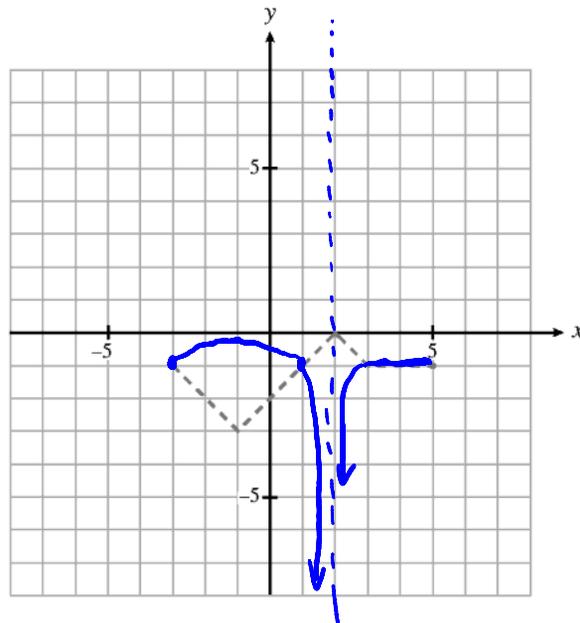
(3 marks)

24
abs
vert refl



2. On the grid provided, sketch the graph of $y = \frac{1}{f(x)}$.

(2 marks)



3. In a population of moths, 78 moths increase to 1000 moths in 40 weeks. What is the doubling time for this population of moths?

(Solve algebraically using logarithms. Answer accurate to at least 2 decimal places.)

(5 marks)

$$A = A_0 c^{t/p}$$

$$\frac{1000}{78} = \frac{78}{78} (2)^{40/p}$$

$$\log 12.8205 = \log 2$$

$$\frac{\log 12.8205}{1} = \frac{40 \log 2}{p}$$

$$p = \frac{40 \log 2}{\log 12.8205}$$

$$p = 10.87 \text{ weeks}$$

4. It is known that 15% of teenagers ride skateboards. Three teenagers are chosen at random. Given that at least one of the three teenagers chosen rides a skateboard, determine the probability that all three teenagers ride skateboards.

(Answer accurate to at least 4 decimal places.)

(5 marks)

$$P(\text{all 3} \mid \text{at least 1})$$

$$= \frac{P(\text{all 3 and at least 1})}{P(\text{at least 1})}$$

$$= \frac{\text{binompdf}(3, .15, 3)}{1 - P(\text{None})}$$

$\begin{matrix} 1 & 2 & 3 \\ \checkmark & \checkmark & \textcircled{3} \\ & & \checkmark \end{matrix}$

$$= \frac{.003375}{1 - \text{binompdf}(3, .15, 0)}$$

$$= \frac{.003375}{.385875} = \frac{3}{343} \text{ or } 0.0087$$

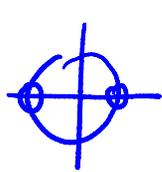
5. Solve algebraically, giving exact values for x , where $0 < x < 2\pi$.

$$2\sin^2 x + \sqrt{3}\sin x = 0$$

(3 marks)

$$\sin x (2\sin x + \sqrt{3}) = 0$$

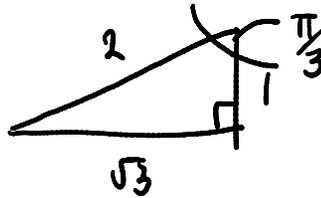
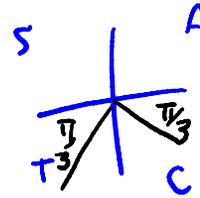
$$\sin x = 0$$



$$x_1 = 0$$

$$x_2 = \pi$$

$$\sin x = -\frac{\sqrt{3}}{2}$$



$$x_3 = \frac{4\pi}{3}$$

$$x_4 = \frac{5\pi}{3}$$

6. The smallest positive solution of $\tan 3x = 0.6$ is $x = 0.18$. Determine the general solution for $\tan 3x = 0.6$. (1 mark)

$$x = 0.18 + \frac{\pi n}{3}, \quad n \in \mathbb{I}$$

7. Prove the identity:

(5 marks)

$$\frac{(1+\sin\theta)}{(1+\sin\theta)} \frac{\sin\theta}{1-\sin\theta} + \frac{\sin\theta}{1+\sin\theta} \frac{(1-\sin\theta)}{(1-\sin\theta)} = \sin 2\theta \sec^2 \theta$$

LEFT SIDE

RIGHT SIDE

$$\begin{aligned} & \frac{\sin\theta(1+\sin\theta) + \sin\theta(1-\sin\theta)}{(1-\sin\theta)(1+\sin\theta)} \\ &= \frac{\sin\theta + \cancel{\sin^2\theta} + \sin\theta - \cancel{\sin^2\theta}}{1 - \sin^2\theta} \\ &= \frac{2\sin\theta}{\cos^2\theta} \end{aligned}$$

$$\begin{aligned} & 2\sin\theta \cancel{\cos\theta} \cdot \frac{1}{\cancel{\cos\theta}^2} \\ &= \frac{2\sin\theta}{\cos^2\theta} \end{aligned}$$

Q. E. D.

END OF EXAMINATION

Examination Rules

1. The time allotted for this examination is two hours.
You may, however, take up to 60 minutes of additional time to finish.
2. Answers entered in the Examination Booklet will not be marked.
3. Cheating on an examination will result in a mark of zero. The Ministry of Education considers cheating to have occurred if students break any of the following rules:
 - Students must not be in possession of or have used any secure examination materials prior to the examination session.
 - Students must not communicate with other students during the examination.
 - Students must not give or receive assistance of any kind in answering an examination question during an examination, including allowing one's paper to be viewed by others or copying answers from another student's paper.
 - Students must not possess any book, paper or item that might assist in writing an examination, including a dictionary or piece of electronic equipment, that is not specifically authorized for the examination by ministry policy.
 - Students must not copy, plagiarize or present as one's own, work done by any other person.
 - Students must immediately follow the invigilator's order to stop writing at the end of the examination time and must not alter an Examination Booklet, Response Booklet or Answer Sheet after the invigilator has asked students to hand in examination papers.
 - Students must not remove any piece of the examination materials from the examination room, including work pages.
4. The use of inappropriate language or content may result in a mark of zero being awarded.
5. Upon completion of the examination, return all examination materials to the supervising invigilator.