Principles of Math 12 **Name:** Key

Unit 2: Logarithms (17%) and Sequences (8%)

1. What is one-half of 2^{20}?

$\frac{1}{2} \times 2^{20} = 2^{19}$

A. 2^{10} B. 1^{20} C. 2^{19} D. 1^{10}

2. If $\log_3 y = c - \log_3 x$, where y>0 and x>0, then y is equal to

A. c-x B. c/x C. $\frac{c^3}{x}$ D. $\frac{3^c}{x}$

$\log_3 y + \log_3 x = c$

$\log_3 (xy) = c$

$xy = 3^c$

$y = \frac{3^c}{x}$

3. The sum of the infinite geometric series $\frac{1}{4} + \frac{1}{16} + \frac{1}{64} \ldots$ is

A. 21/64 B. 1/3
C. 1/2 D. 1

$S = \frac{a}{1-r} = \frac{1/4}{1 - 1/4} = \frac{\frac{1}{4}}{3/4} = \frac{1}{4} \times \frac{4}{3} = 1/3$

4. An investment of $1 000 is earning 4$ interest per annum compounded annually. If the value, V, of the investment after t years is given by $V = 1000(1.04)^t$, then t, written as a function of V, is

A. $t = \frac{\log(V)}{3} - \log(1.04)$ B. $t = \frac{\log(V)}{3\log(1.04)}$

C. $t = \log(V) - 3 - \log(1.04)$ D. $t = \frac{\log(V) - 3}{\log(1.04)}$

$\frac{V}{1000} = 1.04^t$

$\log\left(\frac{V}{1000}\right) = t \log(1.04)$

get t by itself

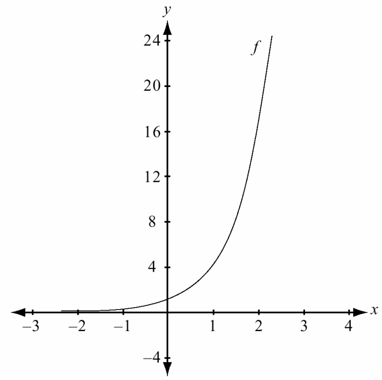
$t = \frac{\log\left(\frac{V}{1000}\right)}{\log 1.04} = \frac{\log V - \log 1000}{\log 1.04}$

$\boxed{t = \frac{\log V - 3}{\log 1.04}}$

5.

Use the following information to answer the next question.

The partial graph of the exponential function $f(x) = 4^x$ is shown below.



The domain of the inverse function f^{-1} is   the range of the original

A. $x > 0,\ x \in R$ $x > 0$

B. $x < 0,\ x \in R$

C. $x \geq 0,\ x \in R$

D. $x \in R$

6. The population of a city was 173 500 on January 1, 1978, and it was 294 000 on January 1, 1992. If the growth rate of the city can be modelled as an exponential function, then the average annual growth rate of the city, expressed to the nearest tenth of a percentage, was

14 years

A. 1.0%   B. 3.8%

C. 6.9%   D. 12.1%

$A = A_0 (c)^{t/p}$

$294{,}000 = 173500\,(c)^{14}$

$c^{14} = 1.6945$

$c = \sqrt[14]{1.6945}$

$c = 1.0384$

growth rate: 3.8 %

7. The price of a particular product doubles every 35 years. If the price of the product was \$16.40 on January 1, 1996, then the price of the product will be \$36.50 in the year

$A = A_0\, c^{t/p}$

A. 2028   B. 2031

C. 2036   D. 2040

$36.50 = 16.40\,(2)^{t/35}$

$2.2256 = 2^{t/35}$

$\log(2.2256) = \frac{t}{35}\log 2$

$t = \frac{35\log(2.2256)}{\log 2}$

$= 40.4$ y

$1996 + 40.4 = 2036$

8. Find the intersection point(s) of $y_1 = (\frac{1}{2})^{x-1}$ and $y_2 = 3^x$. Solve algebraically, and then check your answer using your graphing calculator.

$3^x = (.5)^{x-1}$

$x \log 3 = (x-1) \log .5$

$x \log 3 = x \log .5 - \log .5$

$x \log 3 - x \log .5 = -\log .5$

$x(\log 3 - \log .5) = -\log .5$

$x = \frac{-\log .5}{\log 3 - \log .5} = 0.387$

Check: graph $y_1 = (\frac{1}{2})^{x-1}$

$y_2 = 3^x$

find intersection

$y = 1.53$

9. A new coal mine produced 8 Mt (megatonnes) of coal in 1997, 7 Mt in 1998, and 6.125 Mt in 1999. If this geometric pattern were to continue indefinitely, then the total mass of coal produced by this mine would approach

$r = 7/8 = 0.875$

$a = 8$

A. 21.125 Mt
B. 64 Mt
C. 156.9 Mt
D. an infinite mass

$S = \frac{a}{1-r} = \frac{8}{1 - 7/8} = \frac{8}{1/8} = 8 \times \frac{8}{1} = 64$

10. If $\log_7 m = -\frac{2}{3}$, then the value of m, correct to the nearest hundredth, is 0.27.

$m = 7^{-2/3} = 0.27$

11. In 1996, a particular car was valued at 27 500 and its value decreased exponentially each year afterward. For each of the first 7 years, the value of the car decreased by 24% of the previous year's value. If t is the number of years and v is the value of the car, then the equation for the car's value when $t \leq 7$ is

A. $v=27500(1.76)^t$
B. $v=27500(1.24)^t$
C. $v=27500(0.76)^t$
D. $v=27500(0.24)^t$

12. The x-intercept of the graph of $y = \log_b x$, where b>0 and b≠1, is

A. 0
B. 1
C. undefined
D. dependent on the value of b

let $y=0 \rightarrow \log_b x = 0 \rightarrow x = b^0$

13. A clothing store is going out of business. The owner reduces the cost of each item by 10% of the current price at the start of each week. A jacket costs $120.00 during the 1st week of the sale. If this jacket is still in the store during the 5th week of the sale, then the price of the jacket, to the nearest cent, will be

$t = a \cdot r^{n-1}$

$r = .9$
$a = 120$
$n = 5$

A. $70.00
B. $70.86
C. $78.73
D. $80.00

$t = 120(.9)^4$

14. The relationship between the length and mass of a particular species of snakes is

$$\log m = \log a + 3\log l$$

where a is a given constant, m is the mass of the snake in grams, and l is the length of the snake in metres. This relationship can also be written as

A. $\log(\frac{m}{al^3}) = 0$
B. $\log(\frac{m}{3al}) = 0$
C. $\log(\frac{am}{l^3}) = 0$
D. $\log(\frac{am}{3l}) = 0$

$\log m = \log a + \log l^3$

$\log m - \log a - \log l^3 = 0$

$\log\left(\frac{m}{a l^3}\right) = 0$

15. The sum of the first 10 terms of the geometric sequence -4, 6, -9 ... , to the nearest tenth, is

$n = 10$
$a = -4$
$r = -\frac{6}{4} = -\frac{3}{2}$

A. 153.8
B. 90.7
C. -61.5
D. -453.3

$S_{10} = \frac{a(1-r^n)}{1-r} = \frac{-4(1-(-\frac{3}{2})^{10})}{(1-(-\frac{3}{2}))}$

$= 90.66$

16. If $\log_x(\frac{1}{64}) = -\frac{3}{2}$, then x is equal to

A. 16 B. 8

C. 1/8 D. 1/16

$x^{-3/2} = \frac{1}{64}$

$\left(x^{-3/2}\right)^{-2/3} = \left(\frac{1}{64}\right)^{-2/3}$ → $x = 16$

17. If $\log_3 x = 15$, then $\log_3(\frac{1}{3}x)$ is equal to

A. 14 B. 12

C. 5 D. -15

$\log_3\left(\frac{1}{3}x\right) = \log_3\left(\frac{1}{3}\right) + \log_3 x$

$= -1 + 15 = 14$

18. The equation $y = 4^{3x}$ can also be written as

A. $y = \frac{\log_3 x}{4}$ B. $y = \frac{\log_4 x}{3}$

C. $x = \frac{\log_3 y}{4}$ D. $x = \frac{\log_4 y}{3}$

$\log y = 3x \log 4$

$x = \frac{\log y}{3 \log 4} = \frac{\log_4 y}{3}$ ← base change law

19. Three sums obtained from a particular infinite geometric sequence are $S_1=10$, $S_2=15$, and $S_3=17.5$. The sum of this entire infinite sequence is ___________________.

$S_1 = a = 10$

$S_2 = t_2 + a = 15 \rightarrow t_2 = 5$

$\therefore a = 10$

$r = 5/10 = .5$

$S = \frac{a}{1-r} = \frac{10}{1-.5} = \frac{10}{1/2} = \boxed{20}$

20. The value of $\sum_{n=1}^{10} (\log_2 8)^n$ is

A. 165
C. 88 572

B. 59 049
D. 1 398 100

$\sum_{n=1}^{10} (\log_2 8)^n = \sum_{n=1}^{10} (3)^n = 3 + 9 + 27 + \ldots$

$n = 10$ $r = 3$
$a = 3$

$S_{10} = \frac{3(1-3^{10})}{1-3} = 88{,}572$

21. If $\log_3(2x - y) = 2$ and $\log_2(x + 2y) = 5$, then the value of y, correct to the nearest tenth, is

A. -14.6
C. 11.0

B. -8.2
D. 18.3

Tough question!

$\log_3(2x-y) = 2 \longrightarrow 2x - y = 9$ ①

Hey! This is a system, like math 11!

$\log_2(x+2y) = 5 \longrightarrow x + 2y = 32$ ②

Equation ① can be written: $2x - 9 = y$, sub into ②

$x + 2(2x-9) = 32$

$x + 4x - 18 = 32$

$5x = 50$

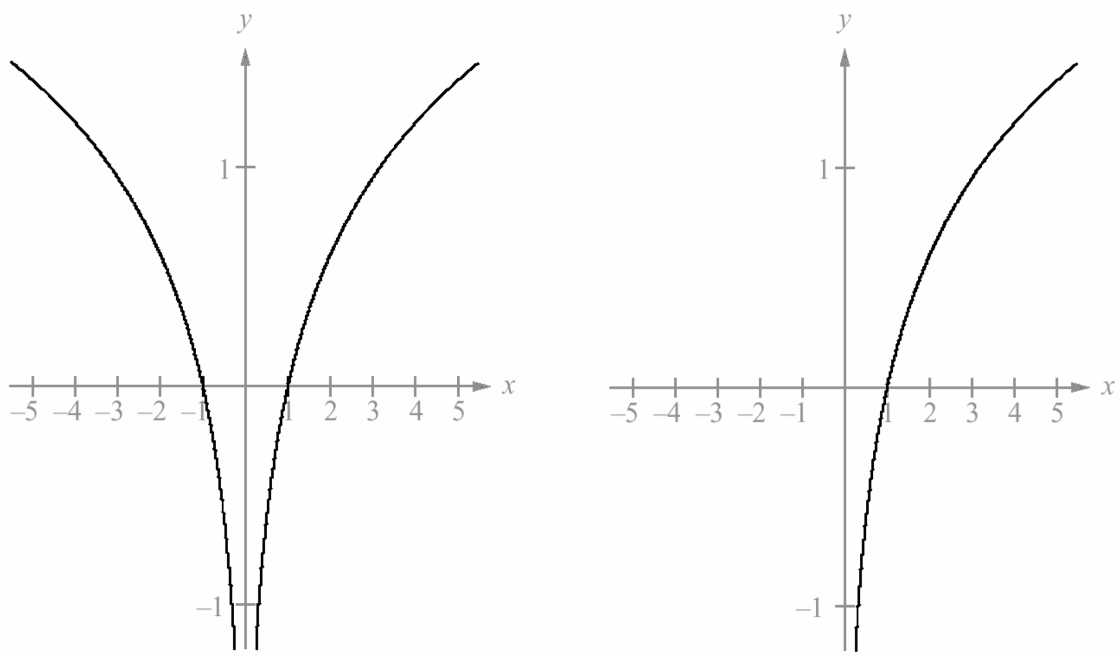
$\boxed{x = 10}$ $\boxed{y = 2x - 9 = 11}$

22.

Use the following information to answer the next question.

A student used a graphing calculator to illustrate identities. The student assumed that $\log(x^2) = 2\log x$ because $\log_a(M^n) = n\log_a M$. The student graphed $y = \log(x^2)$ and obtained the graph shown below on the left. The student then graphed $y = 2\log x$ and obtained the graph shown below on the right.

Graph of $y = \log(x^2)$ **Graph of $y = 2\log x$**



The student realized that the reason why the graphs are not identical is that

A. $\log 0$ is not defined

B. where $x < 0$, $\log(x^2)$ is defined and $\log x$ is not defined

C. where $x < 0$, $\log x$ is defined and $\log(x^2)$ is not defined

D. the range of $y = \log(x^2)$ is different from the range of $y = 2\log x$

23. The expression $\log_5 25^k$ is equal to

$\log_5 25^k = k \log_5 25$

$= k \cdot 2$

A. 2k
B. k^2
C. 25k
D. 25^k

24. The atmospheric pressure P, in kilopascals (kPa), at a distance d, in kilometres above Earth, is given by the formula $P = 100(10^{-0.0542d})$. A particular plane is designed to fly safely when the atmospheric pressure is greater than 15 kPa. The plan can fly safely to a maximum height of

A. 15.5 km
B. 15.2 km
C. 10.9 km
D. 2.8 km

$15 = 100(10^{-.0542d})$

$.15 = 10^{-.0542d}$

$\log(.15) = -.0542d \rightarrow d = \dfrac{\log(.15)}{-.0542} = 15.2$

25. The graph of $y = a^{x-2}$ has a y-intercept of ← let x = 0

A. $\dfrac{1}{a^2}$
B. $-\dfrac{1}{a^2}$
C. $-a^2$
D. 2

$y = a^{0-2} \rightarrow y = a^{-2} = \dfrac{1}{a^2}$

26. The point (64, 4) lies on the graph of y=$\log_b$(x). If the point (2, k) lies on the graph of y=b^x, then the value of k is

sub into

A. 8
B. 16
C. 32
D. 256

$\log_b 64 = 4$

$b^4 = 64$

$b = \sqrt[4]{64}$ or $64^{1/4}$

$y = b^x$

$y = (64^{1/4})^x$ where x = 2

$y = (64^{1/4})^2 = 64^{1/2} = \underline{8}$

k = 8

27. If $\log_b a = 0.82$, then the value of $\log_b(\frac{b}{a})$, correct to the nearest hundredth, is __________.

$$\log_b\left(\frac{b}{a}\right) = \log_b b - \log_b a$$

$$= 1 - .82 = .18$$

28. Given that $\log_b 64 = 3/2$, the value of b is

$\log_b 64 = 3/2 \rightarrow b^{3/2} = 64$

$(b^{3/2})^{2/3} = 64^{2/3}$

$b = \sqrt[3]{64}^{\,2} = 4^2 = 16$

A. 16

B. 42 2/3

C. 96

D. 512

29. If $\log_5(125x) = 25$, then the value of x is

A. 5^{28}

B. 5^{22}

C. $5^{\frac{25}{3}}$

D. 5

$125x = 5^{25}$

$5^3 \cdot x = 5^{25}$

$x = \frac{5^{25}}{5^{-3}} = 5^{22}$

30. A student wants to use a graphing calculator to graph $y = \log_5 x$. If the calculator accepts only common logarithms, then an equivalent equation that could be used to obtain the graph is

A. $y = \frac{\log x}{\log 5}$

Base change Rule!

B. $y = \log x - \log 5$

C. $y = 5\log x$

D. $y = \frac{\log x}{5}$

31. The half-life of phosphorus-32 is 14.3 days. The length of time that it will take 96.2 g of phosphorus-32 to decay to 12.5 g, to the nearest day, is

$A = A_0 (r)^{t/p}$

A. 8 days

B. 26 days

C. 42 days

D. 52 days

$t/14.3$

$$\frac{12.5}{96.2} = \frac{96.2}{96.2}\left(\frac{1}{2}\right)$$

$$.13 = \left(\frac{1}{2}\right)^{t/14.3}$$

$$\log(.13) = \frac{t}{14.3}\log\left(\frac{1}{2}\right)$$

$$t = \frac{14.3\log(.13)}{\log\left(\frac{1}{2}\right)}$$

$$t = 42.1$$

32. If $\log_2 b = c$, then $\log_4 b$ equals

hmm.. ahah! Try base change!

A. c/2

B. c^2

C. 2c

D. $\sqrt{c}$

$$\log_4 b = \frac{\log_2 b}{\log_2 4} = \frac{\log_2 b}{2} = \frac{c}{2}$$

33. In a geometric sequence, the first term is 2/81 and the sixth term is 3/16. The common ratio for this geometric sequence is

A. 2/3

B. 3/2

C. 2

D. 3

$$\underline{2/81},\ \underset{r}{\underline{\quad}},\ \underset{r}{\underline{\quad}},\ \underset{r}{\underline{\quad}},\ \underset{r}{\underline{\quad}},\ \underset{r}{\underline{3/16}}$$

$$\frac{2}{81}r^5 = \frac{3}{16}$$

$$r^5 = \frac{243}{32}$$

$$r = \sqrt[5]{243/32} = 3/2$$

34. If $\log_x \frac{125}{27} = -\frac{3}{2}$, then the value of x is $\rightarrow$ $x^{-3/2} = \frac{125}{27}$

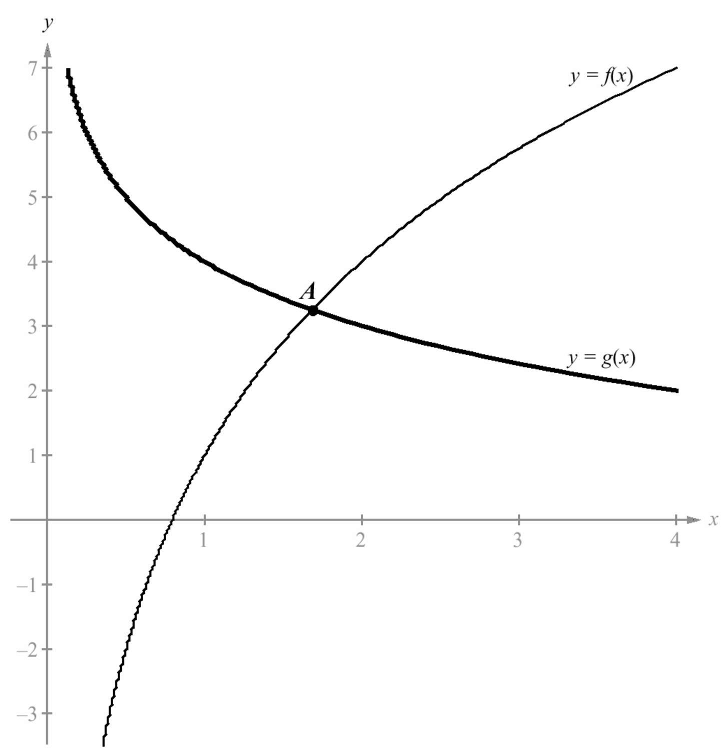
A. 9/25
B. 25/9
C. -9/25
D. -25/9

$$\left(x^{-3/2}\right)^{-2/3} = \left(\frac{125}{27}\right)^{-2/3} \qquad x = \left(\frac{125}{27}\right)^{-2/3} = \boxed{\frac{9}{25}}$$

35.

Use the following information to answer the next question.

The partial graphs of $f(x) = 1 + 3\log_2 x$ and $g(x) = 4 - \log_2 x$ are shown below. The graphs intersect at point A.



Based on the information above, the x-coordinate of point A can be determined by solving

A. $3\log_2 x + 1 = 0$

B. $4 - \log_2 x = 0$

C. $\log_2 x = \frac{5}{2}$

D. $\log_2 x = \frac{3}{4}$

To solve:

$1 + 3\log_2 x = 4 - \log_2 x$

$-4 \quad +\log_2 x \qquad -4 \quad +\log_2 x$

$-3 + 4\log_2 x = 0$

$4\log_2 x = 3$

$\log_2 x = 3/4$

36. 0

A mathematics class was asked to solve the equation

$$3^{x+2} = 6^x$$

The attempts of two students to solve the equation are shown below. Each student made one error that led to an incorrect solution.

Student A	**Student B**
$3^{x+2} = 6^x$	$3^{x+2} = 6^x$
$3^{x+2} = 3^{2x}$	$\log 3^{x+2} = \log 6^x$
$x + 2 = 2x$	$(x + 2) \log 3 = x \log 6$
$2 = 2x - x$	$2 \log 3 = -x + x \log 6$
$x = 2$	$2 \log 3 = x(-1 + \log 6)$
	$\dfrac{2\log3}{(-1+\log6)} = x$
	x is approximately -4.3

Arrgghhh!

sloppy! Arrgghh!

a) Show that neither x=2 nor x= -4.3 satisfies the original equation. Well, they don't!

b) Identify the error that was made by each student and state why each error leads to an incorrect answer. see above.

c) Solve the equation correctly. Give your answer to 2 decimal places, if necessary.

$3^{x+2} = 6^x$

$\log(3^{x+2}) = \log 6^x$

$(x+2)\log 3 = x \log 6$

$x\log 3 + 2\log 3 = x \log 6$

$2\log 3 = x \log 6 - x\log 3$

$2\log 3 = x(\log 6 - \log 3)$

$x = \dfrac{2\log 3}{\log 6 - \log 3} = 3.17$

37. In a nuclear disaster at Chernobyl in April 1986, approximately 12 600 kg of radioactive iodine-131 was released into the atmosphere. The half-life of iodine-131 is 8.04 days; therefore, after 8.04 days, half of the iodine-131 had decayed. The amount, N(t), of iodine-131, in kg, remaining after t days is given by

$$N(t) = 12600(\frac{1}{2})^{\frac{t}{8.04}}$$

$N = 12600\left(\frac{1}{2}\right)^{30/8.04}$

The approximate mass of iodine-131 remaining after 30 days was

A. 70 kg
B. 131 kg
C. 420 kg
D. 949 kg

38. The expression $2\log_a 5 + \log_a 6 - \frac{1}{3}\log_a 8$, a>0, equals

A. $\log_a 29$
B. $\log_a 30$
C. $\log_a 75$
D. $\frac{8}{3}\log_a 3$

$= \log_a 5^2 + \log_a 6 - \log_a 8^{1/3}$

$= \log_a \left(\frac{25 \cdot 6}{2}\right) = \log_a 75$

39. A logarithmic form of $81^{\frac{3}{4}} = 27$ is

A. $\log_{27}(\frac{3}{4}) = 81$
B. $\log_{\frac{3}{4}}(27) = 81$
C. $\log_{27}(81) = \frac{3}{4}$
D. $\log_{81}(27) = \frac{3}{4}$

40. If $\log_2 x + 8 = 0$, then the value of x is

A. -3
B. -1/256
C. 1/256
D. 3

$\log_2 x = -8$

$x = 2^{-8} = \frac{1}{256}$

41. The graph of y=$\log_4$(x), where x>0, lies entirely

A. above the x-axis
B. below the x-axis
C. to the left of a the y-axis
D. to the right of the y-axis

42. The value of $\sum_{n=1}^{9}\log(\frac{n}{n+1})$ is equal to

A. -1

B. $\log\frac{45}{54}$

C. $\log\frac{35638}{5040}$

D. $-1+\log 9$

$= \log\left(\frac{1}{2}\right) + \log\left(\frac{2}{3}\right) + \log\left(\frac{3}{4}\right) + \ldots \log\left(\frac{9}{10}\right)$

$= \log\left(\frac{1\cdot\cancel{2}\cdot\cancel{3}\cdot\cancel{4}\cdot\cancel{5}\cdot\cancel{6}\cdot\cancel{7}\cdot\cancel{8}\cdot\cancel{9}}{\cancel{2}\cdot\cancel{3}\cdot\cancel{4}\cdot\cancel{5}\cdot\cancel{6}\cdot\cancel{7}\cdot\cancel{8}\cdot\cancel{9}\cdot 10}\right) = \log\left(\frac{1}{10}\right) = -1$

43. In the equation $\log_{(x+2)} 5 = 10$, the value of x, correct to the nearest hundredth, is

A. -0.42

B. -0.83

C. -1.41

D. -1.50

$(x+2)^{10} = 5$

$x+2 = \sqrt[10]{5}$

$x = \sqrt[10]{5} - 2 =$

44. The point P(-1, 1/3) lies on the graph of the exponential function f(x) = b^x. The value of the base, b, of the exponential function, f, is

A. 1/3

B. 3

C. -1/3

D. -3

$b^{-1} = \frac{1}{3} \rightarrow \frac{1}{b} = \frac{1}{3} \quad b = 3$

45. The first term of a geometric sequence is 16, and the common ratio is -1/2. The sum of the first 7 terms of this sequence is

A. 114

B. 31.75

C. 10.75

D. 0.25

$S_7 = \frac{16\left(1-\left(-\frac{1}{2}\right)^7\right)}{1-\left(-\frac{1}{2}\right)} =$

46. The value of $\sum_{k=5}^{15}(2^{k-1})$ is $= 2^4 + 2^5 + \ldots 2^{14}$

$a = 16$

$r = 2$

$n = 15 - 5 + 1 = 11$

A. 16 368

B. 16 383

C. 32 752

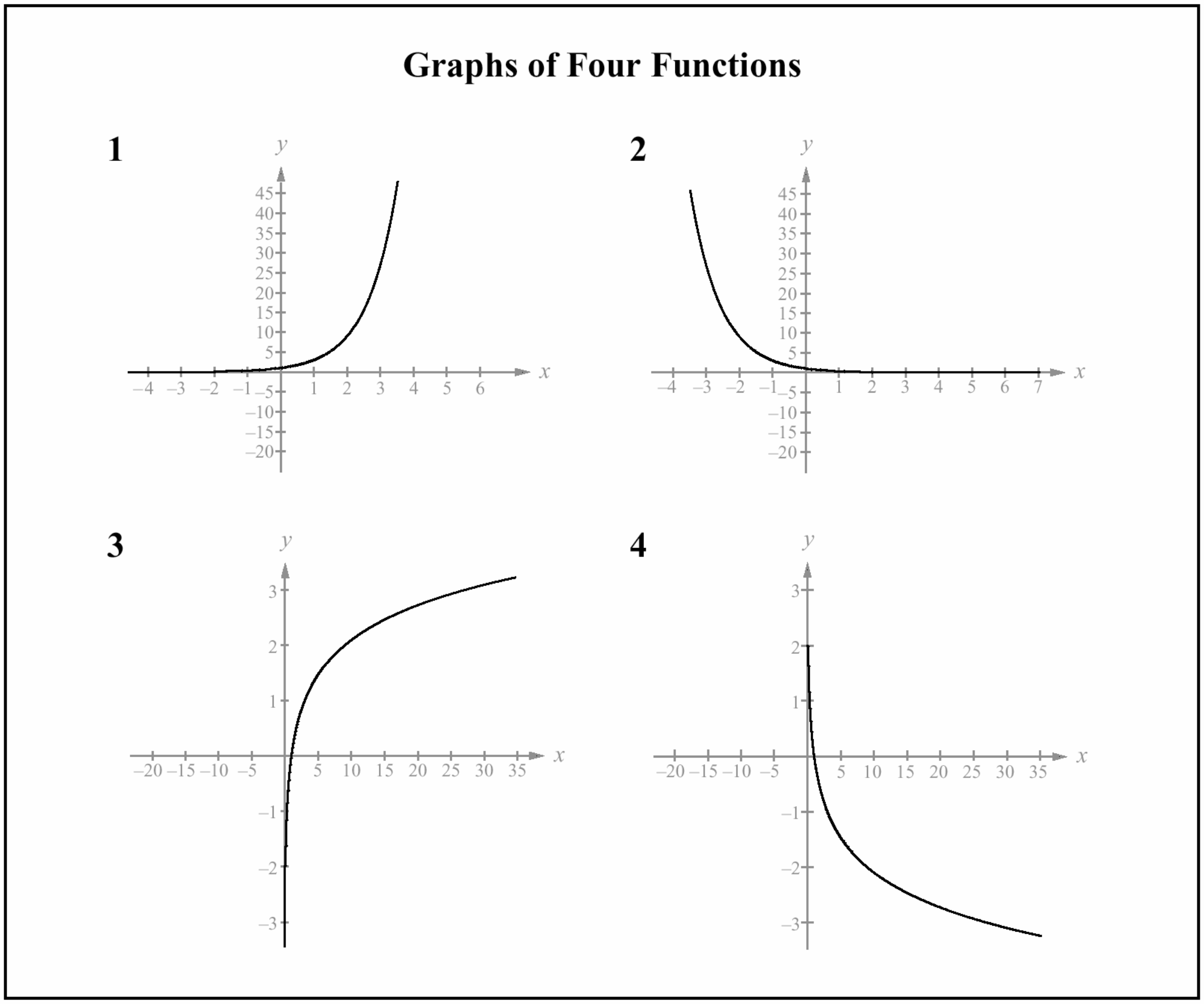
D. 32 767

$$S_{11} = \frac{16(1-2^{11})}{1-2} = 32752$$

47. How many terms are there in the series from question 46?

$15 - 5 + 1 = 11$

48.



Match each of the graphs, as numbered above, to the statement below that describes is.

a) Graph #__2__ represents an exponential function with a base between 0 and 1.
b) Graph #__1__ represents an exponential function with a base greater than 1.
c) Graph #__4__ represents a logarithmic function with a base between 0 and 1.
d) Graph #__3__ represents a logarithmic function with a base greater than 1.

49. Give an expression for the number of terms in the series: $\sum_{k=x}^{y} 5(1.4)^{k-2}$

A. y
B. y-x
C. y-x + 1
D. y-x - 1

y - x + 1

50. Determine the common ratio of the infinite geometric sequence:

$$\log_2 a, \log_8 a, \log_{512} a \quad , \ldots \text{ where } a > 0$$

A. 1/3
B. 1/2
C. 3
D. 2

base change

$$\frac{\log_8 a}{\log_2 a} = \frac{\frac{\log_2 a}{\log_2 8}}{\log_2 a} = \frac{\frac{\log_2 a}{3}}{\log_2 a} = \frac{\cancel{\log_2 a}}{3} \times \frac{1}{\cancel{\log_2 a}} = 1/3$$

51. An earthquake in Vancouver measured 3.2 on the Richter scale. An earthquake in Tokyo measure 6.3 on the Richter scale. How many times as intense was the earthquake in Tokyo compared to the earthquake in Vancouver?

A. 3.10
~~B. 1.97~~
C. 93.01
D. 1258.93

$10^{6.3-3.2}$ = how many times

$10^{3.1} = 1258.93$

52. An earthquake in Turkey is measured at 5.2 on the Richter scale. It is 2500 times stronger than an earthquake near Victoria. What is the measurement of the earthquake in Victoria?

A. 1.80
B. 1.63
C. 3.61
D. 2.70

$10^{5.2-x} = 2500$

$\log 10^{5.2-x} = \log 2500$

$(5.2-x)\cancel{\log} 10 = \log 2500$

$5.2 - x = \log 2500$

$5.2 - \log 2500 = x$

$x = 1.80$

53. Sound coming from a stereo speaker is measured at 124 dB. How many times stronger is this than the sound of a hair dryer, measured at 104 dB?

A. 10 times stronger
B. 100 times stronger
C. 200 times stronger
D. 16 times stronger

$10^{\frac{124-104}{10}}$ = how many times

$10^2 = \underline{100}$

54. A certain radioactive element has a decay formula given by $A = A_0(0.75)^t$. Rewrite this formula with a base of 0.5 instead of 0.75.

A. $A = A_0(0.5)^t$
B. $A = A_0(0.75)^{\frac{t}{2}}$
C. $A = A_0(0.5)^{0.415t}$
D. $A = A_0(0.5)^{0.75t}$

$0.5^k = .75$

$k \log .5 = \log .75$

$k = \frac{\log .75}{\log .5} = .415$

$A = A_0 (.5)^{.415t}$

55. Rewrite the above formula as a continous growth equation, with base *e*.

$e^k = .75$

$k \ln e = \ln .75$

$k = \ln .75$

$A = A_0 e^{t \ln .75}$

or $A = A_0 e^{-.288t}$

56. Solve for x: log (5-x) + log (5+x) = log(16)

A. x=3
B. x= -3
C. $x = \pm 3$
D. no solution

$\log\left[(5-x)(5+x)\right] = \log 16$

$(5-x)(5+x) = 16$

$25 - x^2 = 16$

$0 = x^2 - 9$

$(x-3)(x+3) = 0$

$x = +3, -3$

$5-x > 0 \rightarrow 5 > x$

57. Determine the domain of the function $y=\log_x (5-x)$.

$x > 0$

$x \neq 1$

A. x<5
B. x<5, x≠1
C. 0< x < 5
D. 0 < x < 5, x≠1

58. Solve agebraically: $2 \log_4 (x) - \log_4 (x + 3) = 1$

$\log_4 x^2 - \log_4 (x+3) = 1$

$\log_4\left[\dfrac{x^2}{x+3}\right] = 1$

$\dfrac{x^2}{x+3} = 4$

$x^2 = 4(x+3)$

$x^2 = 4x + 12$

$x^2 - 4x - 12 = 0$

$(x-6)(x+2) = 0$

$x = 6, \cancel{-2}$

59. Evaluate $\log_2 \sqrt{8}$ $= \log_2 2^{3/2} = \frac{3}{2} \log_2 2 = 3/2$

A. $\sqrt{2}$

B. 3/2

C. 3

D. 8

60. Which expression is equivalent to $\log_5 30$?

A. log 6

B. log 30 – log 5

C. $\frac{\log 5}{\log 30}$

D. $\frac{\log_\pi 30}{\log_\pi 5}$

base change!

$$\frac{\log_\pi 30}{\log_\pi 5}$$

61. Express $\frac{1}{2}\log a - \log b - \log c$ as a single logarithm.

A. $\log(\frac{\sqrt{a}}{bc})$

B. $\log(\frac{c\sqrt{a}}{b})$

C. $\log(\frac{1}{2}a - b - c)$

D. $\log(\frac{a}{2bc})$

$\log \sqrt{a} - \log b - \log c = \log\left(\frac{\sqrt{a}}{bc}\right)$

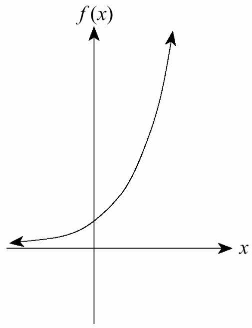
typo

$f(x) = 2^{-x}$?

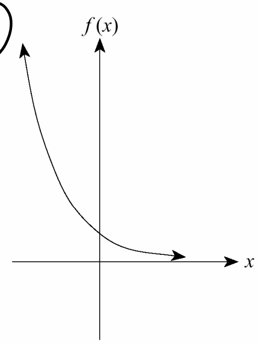
62.

Which graph best represents the function ~~$f(x) = 2^{\pm x}$?~~

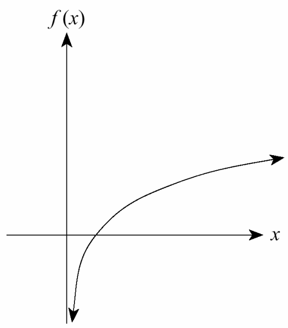
A.



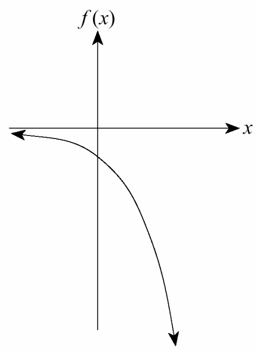
B.



C.



D.



63. Solve for x: $\log_2[\log_x(\log_3 9)] = -1$

A. 2

B. 3

C. 4

D. 5

$\log_2(\log_x(2)] = -1$

$\log_x 2 = 2^{-1}$

$\log_x 2 = \frac{1}{2}$

$(x^{\frac{1}{2}})^2 = 2^2 \rightarrow x = 4$

64. Determine a single geometric mean between 9 and 25.

A. $\sqrt{34}$

B. 15

C. 16

D. 17

$9, \_\_, 25$

$r \quad r$

$9r^2 = 25$

$r^2 = 25/9$

$r = \pm 5/3$

$9, \left(9 \times \frac{5}{3}\right), 25$

$\boxed{15}$

65. If the sum of an infinite geometric series is 9 and the first term is 6, determine the common ratio.

A. -1/3
B. 1/3
C. 2/3
D. 3/2

$S = \frac{a}{1-r} \rightarrow 9 = \frac{6}{1-r} \rightarrow 9(1-r) = 6 \rightarrow 9 - 9r = 6$

$-9r = -3$

$r = 3/9 = 1/3$

66. What restrictions for x exist for the equation $\log_x x + \log_x (2-x) = 5$?

A. $x>0,\ x \neq 1$
B. $x<2$
C. $0 < x < 2$
D. $0 < x < 2,\ x \neq 1$

$2 - x > 0 \rightarrow 2 > x$ also $x \neq 1$, also $x > 0$

67. A city has a population of 15 000 and the population decreases by 8% per year. How many years will it take for the population to become 5 000? (accurate to 1 decimal place).

$A = A_0 (c)^{t/p}$

$5000 = 15000 (.92)^t$

$\frac{1}{3} = .92^t \rightarrow \log\left(\frac{1}{3}\right) = t \log .92$

$t = \frac{\log(1/3)}{\log .92} = 13.2\ y$

68. Determine the logarithmic form of $4^x=3$.

A. $\log_3 x=4$
B. $\log_3 4=x$ ← Easy Marks!
C. $\log_x 4=3$
D. $\log_4 3=x$

69. What is the value of x if $\log_5 x = 4$?

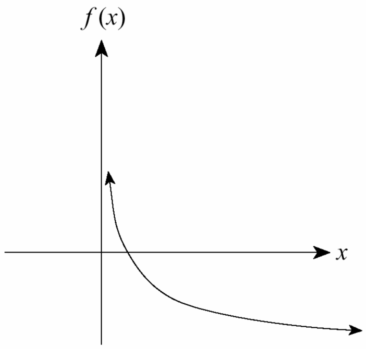
A. $\sqrt[5]{4}$
B. $\sqrt[4]{5}$
C. 5^4
D. 4^5

$x = 5^4$

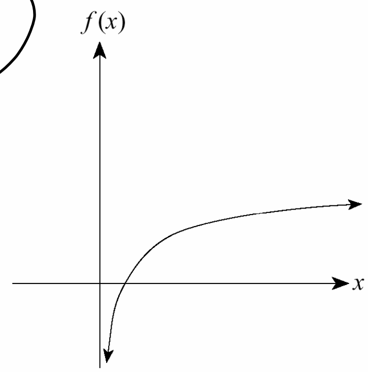
70.

Which graph **best** represents the function $f(x) = \log_3 x$?

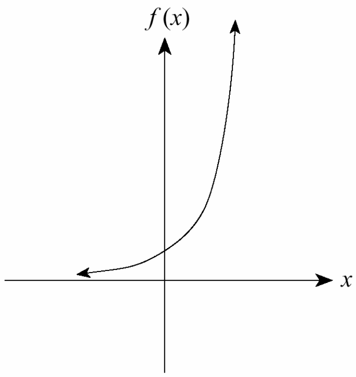
A.



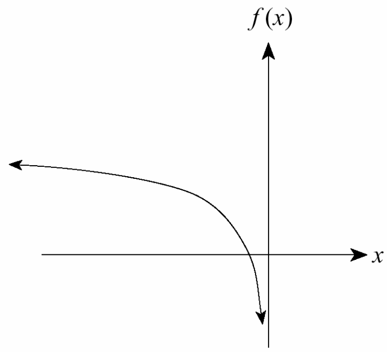
B.



C.



D.



71. Evaluate: $-16\log_2(\frac{1}{8})$ $= -16 \log(2^{-3}) = 16 \cdot -3 \cancel{\log_2} 2 = 48$

A. -4

B. -1/4

C. 0

D. 48

72. If $\log_5 x = 4.26$, what is the value of $\log_5 25x^2$?

A. 2.66

B. 3.80

C. 8.26

D. 10.52

$$\log_5 25x^2 = \log_5 25 + \log_5 x^2 = 2 + 2\log_5 x$$
$$= 2 + 2(4.26) = 10.52$$

73. Simplify: $(\log_x y)(\log_y x)$ hmm... try base change

A. 0

B. 1

C. $xy^{(x+y)}$

D. $\log_{xy}(x+y)$

$$\log_x y \cdot \frac{\log_x x}{\log_x y} = \log_x y \cdot \frac{1}{\log_x y} = 1$$ wow! That is so cool!

74. Determine the values of x ($x \neq 0$) such that the following infinite geometric series has a finite sum.

$$1+\frac{1}{4}x+\frac{1}{16}x^2+\frac{1}{64}x^3+...$$

$r = \frac{1}{4}x$

A. x < ¼
B. x > 4
C. -4 < x < 4
D. -1/4 < x < ¼

$-1 < \frac{1}{4}x < 1$ ← mult everything by 4

$$\boxed{-4 < x < 4}$$

75. Solve for x: $\log_5(2x+1)=1-\log_5(x+2)$

$\log_5(2x+1) + \log_5(x+2) = 1$

$\log_5[(2x+1)(x+2)] = 1$

$(2x+1)(x+2) = 5$

$2x^2 + 5x + 2 = 5$

$2x^2 + 5x - 3 = 0$

$(2x - 1)(x + 3) = 0$

$x = \frac{1}{2}, \cancel{-3}$

76. Evaluate log5(15) (Accurate to 2 decimal places)

$\frac{\log 15}{\log 5} =$

A. 0.70
B. 1.17
C. 10.48
D. 1.68

77. If the graph of y=a^{x} goes through the point (4, 625), determine the value of a.

$625 = a^4$ $a = \sqrt[4]{625} = \pm 5$

A. 1/5
B. 5
C. -5
D. ±5

78. Determine the domain of the function y=log4 (2x+4)

$2x + 4 > 0$
$2x > -4$ → $x > -2$

A. x> 4
B. ~~x < 4~~
C. x > 0
D. x> -2

79. Determine an expression for x, if $\log x = \log a - 2\log b + \frac{1}{4}\log c$.

A. $\dfrac{a}{b^2\sqrt[4]{c}}$

B. $\dfrac{a\sqrt[4]{c}}{b^2}$

C. $\dfrac{a}{c^4\sqrt{b}}$

D. $\dfrac{ac^4}{\sqrt{b}}$

$\log x = \log a - \log b^2 + \log c^{1/4}$

$\log x = \log\left[\dfrac{a\sqrt[4]{c}}{b^2}\right] \rightarrow x = \dfrac{a\sqrt[4]{c}}{b^2}$

80. A radioactive material decays according to the formula $A = A_0 10^{-kt}$, where A is the final amount, A_0 is the initial amount, and t is the time in years. Find *k*, if 600 grams of this material decays to 475 grams in 8 years. (Accurate to 4 decimal places)

$\dfrac{475}{600} = \dfrac{600}{600}(10)^{-8k}$

$\log\left(\dfrac{19}{24}\right) = -8k\,\cancel{\log}\,10 \rightarrow k = \dfrac{\log\left(19/24\right)}{-8}$

$k = .0127$

81. If $a = b^{c\log_b d}$, then which of the following must be true? $\boxed{m^{\log_m n} = n}$ use this

A. a=cd

B. $a=b^c$

C. $a=d^c$

D. a=dc

$a = b^{\log_b(d^c)} \rightarrow a = d^c$

82. Solve: $\pi^{2\log_\pi x + \log_\pi x} = 125$

A. 2

B. 3

C. 5

D. 3.1416

$\pi^{\log_\pi x^2 + \log_\pi x} = 125$

$\pi^{\log_\pi x^3} = 125 \rightarrow x^3 = 125$

$\boxed{x = 5}$

83. Determine the point where the asymptotes of the graphs $y = b^{x-1} + 2$ and $y = \log_b(x-1) + 2$ intersect.

A. (1,1)

B. (1,2)

C. (2,1)

D. (2,2)

$y = b^{x-1} + 2$ has asymptote $y = 2$

$y = \log_b (x-1) + 2$ has asymp. $x = 1$

these cross at (1,2)

84. Determine the number of terms in the series $\sum_{k=7}^{57} (k+1)^2$

A. 49

B. 50

C. 51

D. 52

$57 - 7 + 1 = 51$

85. The sum of the first four terms of a geometric series with a first term of 5 is given by $S_4 = \frac{5(1-3^4)}{1-3}$. Determine the common ratio r.

A. -3

B. 3

C. 4

D. 5

$S_n = \frac{a(1-r^n)}{1-r}$

hmm.... $r = 3$

86. Evaluate: $\sum_{k=3}^{\infty} 12(-\frac{2}{3})^{k-1}$

A. 36/5

B. 16/5

C. 16

D. 36

$a = 16/3 \qquad r = -2/3$

$= 12(-2/3)^2 + 12(-2/3)^3 + \ldots$

$= 16/3 + \frac{-32}{9} + \ldots$

$S_\infty = \frac{a}{1-r} = \frac{16/3}{1 - -2/3} = \frac{16/3}{5/3} = \frac{16}{3} \times \frac{3}{5} = \frac{16}{5}$

Scholarship Questions! Nasty, with big fangs, and sharp teeth! Be careful!

87. Solve: $\log_2(2-x)+\log_2(1-x)=\log_2(x+4)+1$

$\log_2\left[(2-x)(1-x)\right] \quad -\log_4(x+4) = 1$

$\log_2\left[\frac{(2-x)(1-x)}{x+4}\right] = 1 \longrightarrow \frac{(2-x)(1-x)}{x+4} = 2$

$(2-x)(1-x) = 2(x+4)$

$2-3x+x^2 = 2x+8$

$x^2-5x-6=0$

$(x+1)(x-6)=0$

$x=-1, \cancel{6}$

88. The sum of the first three terms of a geometric sequence is -14, while the second term is 12. Find all such sequences.

$S_3 = -14$ $\quad t_2 = 12$

$t_1 + t_2 + t_3 = -14$

$\frac{12}{r} + 12 + 12r = -14$

$r\frac{12}{r} + r12r = r(-26)$

$\frac{12}{2} + \frac{12r^2}{2} = \frac{-26r}{2}$

$t_1 = \frac{12}{r}$ $\quad t_3 = 12r$

$6r^2 + 13r + 6 = 0$

$(3r+2)(2r+3) = 0$

$r_1 = -\frac{2}{3}$ $\quad r_2 = \frac{3}{2}$

Sequence 1: $\frac{12}{r_1}, 12, 12r_1$

$= \frac{12}{-2/3}, 12, 12(-2/3)$

$\boxed{= -18, 12, -8}$

Sequence 2: $\frac{12}{3/2}, 12, 12(3/2)$

$\boxed{= 8, 12, 18}$

89. Solve: $(\log_2 x)^2 - 2\log_2 x - 8 = 0$

let $A = \log_2 x$

$A^2 - 2A - 8 = 0$

$(A-4)(A+2) = 0$

$A = 4$

$\log_2 x = 4 \rightarrow x = 2^4 = 16$

$A = -2$

$\log_2 x = -2 \rightarrow x = 2^{-2} = 1/4$

$\boxed{x = 16 \text{ or } 1/4}$

90. Each side of a square is 50 cm long. A second square is inscribed in the original square by joining the midpoints of the sides of the original square. This process is repeated over and over again.

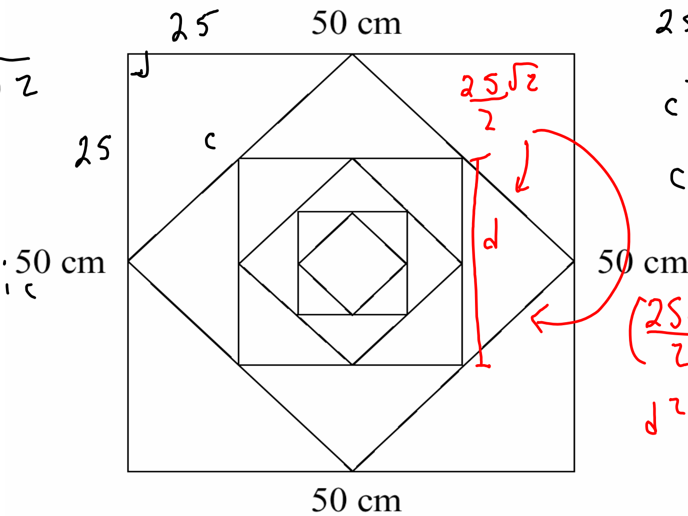
$P_1 = 4 \times 50 = 200$

$P_2 = 4 \times 25\sqrt{2} = 100\sqrt{2}$

$P_3 = 4 \times 25 = 100$

This is a geometric series with

$a = 200, \quad r = \frac{1}{\sqrt{2}}$



$25^2 + 25^2 = c^2$

$c^2 = 1250$

$c = \sqrt{1250} = 25\sqrt{2}$

$\left(\frac{25\sqrt{2}}{2}\right)^2 + \left(\frac{25\sqrt{2}}{2}\right)^2 = d^2$

$d^2 = 625 \quad d = 25$

a) Find the perimeter of the sixth square. (Give an answer that is exact **or** accurate to 2 decimal places).

b) If this process is continued indefinitely, find the sum of the perimeters of **all** the squares. Give your answer accurate to 2 decimal places.

a) $t_n = ar^{n-1} \rightarrow t_6 = 200\left(\frac{1}{\sqrt{2}}\right)^5 = \frac{200}{\sqrt{2}^5} = \frac{200}{4\sqrt{2}} = \frac{50}{\sqrt{2}}$ or 35.36

b) $S = \frac{a}{1-r} \rightarrow S = \frac{200}{1-\left(\frac{1}{\sqrt{2}}\right)} = 682.84$

switch x and y to find $f^{-1}(x)$

91. If $f(x) = 2\log_3(x) + 1$ find $f^{-1}(x)$.

$x = 2\log_3 y + 1$

$x - 1 = 2\log_3 y$

$\frac{x-1}{2} = \log_3 y$

$3^{\frac{x-1}{2}} = y$

$\therefore f^{-1}(x) = 3^{\frac{x-1}{2}}$

92. An infinite geometric series has the property that the sum of any two consecutive terms is equal to the sum of all the terms that follow these two terms.

a) Determine all possible values for the common ratio r.

let series be $a, ar, \underbrace{ar^2, ar^3, ar^4, \ldots}_{\text{infinite sum}}$

$S = \frac{ar^2}{1-r}$

$a + ar = \frac{ar^2}{1-r}$ ← now, divide by a

$1 + r = \frac{r^2}{1-r}$

$(1+r)(1-r) = r^2$

$1 - r^2 = r^2$

$1 = 2r^2$

$\frac{1}{2} = r^2$

$r = \pm\sqrt{\frac{1}{2}} = \frac{\pm 1}{\sqrt{2}}$ or $\pm\frac{\sqrt{2}}{2}$

b) If the fifth term is 8, find the value(s) of the first term.

$t_5 = ar^4$

$8 = a\left(\pm\frac{1}{\sqrt{2}}\right)^4$

$8 = a\left(\frac{1}{4}\right)$

$\boxed{a = 32}$

93. There are two infinite geometric series that have a 4th term of -8 and an 8th term of -128/81. Find the sum of each series.

$$\_\_,\ \_\_,\ \_\_,\ -8,\ \_\_,\ \_\_,\ \_\_,\ \frac{-128}{81}$$

$r \quad r \quad r \quad r$

$-8r^4 = -\frac{128}{81}$

$r^4 = +\frac{128}{648}$

$r = \sqrt[4]{\frac{128}{648}} = \pm \frac{2}{3}$

If $r = 2/3$

$t_4 = -8$

$ar^{n-1} = -8$

$a(2/3)^3 = -8$

$a = -8/(2/3)^3 = -27$

$S = \frac{a}{1-r} = \frac{-27}{1-2/3}$

$\boxed{S_\infty = -81}$

If $r = -2/3$

$a\left(-2/3\right)^3 = -8$

$a = \frac{-8}{(-2/3)^3} = 27$

$S = \frac{a}{1-r} = \frac{27}{1- -2/3}$

$\boxed{S_\infty = 16.2}$

94. Solve: $4(\log_2 x)^4 - 7(\log_2 x)^2 + 3(\log_2 x) = 0$. Give answers that are exact or accurate to two decimal places.

let $A = \log_2 x$

$4A^4 - 7A^2 + 3A = 0$ ← use graphing calc to find roots

$A_1 = -3/2$

$\log_2 x = -3/2$

$x = 2^{-3/2}$

$x = .35$

$A_2 = 0$

$\log_2 x = 0$

$x = 2^0$

$x = 1$

$A_3 = 1/2$

$\log_2 x = 1/2$

$x = 2^{1/2}$

$x = \sqrt{2}$

$A_4 = 1$

$\log_2 x = 1$

$x = 2^1$

$x = 2$

95. A population of rabbits increases by 90% every 6 months. At the present time there are 200 rabbits. How many **years** will it take for the population to reach 1 000 000 rabbits? (Accurate to 2 decimal places.)

$A = A_0 (c)^{t/p}$ $\quad t/6 \quad$ where t = time in months

$1{,}000{,}000 = 200(1.9)^{t/6}$

$5000 = 1.9^{t/6}$

$\log(5000) = \frac{t}{6} \log 1.9$

$t = \frac{6 \log(5000)}{\log 1.9}$

$t = 79.62$ months $= 6.63$ y

96. A geometric sequence has all positive terms. The sum of the first 2 terms of this geometric sequence is 20/3. The sum of the reciprocals of the first 2 terms is 5/8. Find all possibilities for the first term of the sequence.

$t_1 = a \quad t_2 = ar$

① $a + ar = \frac{20}{3}$

$a(1+r) = \frac{20}{3}$

$a = \frac{20}{3(1+r)}$

② $\frac{1}{a} + \frac{1}{ar} = \frac{5}{8}$ ← mult by $8ar$

$8r + 8 = 5ar$

sub into for a

$8r + 8 = 5r\left(\frac{20}{3(1+r)}\right) \rightarrow 8r + 8 = \frac{100r}{3(1+r)}$ cross multiply

$(8r+8)(3)(1+r) = 100r$

$3(8 + 16r + 8r^2) = 100r$

$24 + 48r + 24r^2 = 100r$

$\frac{24r^2}{4} - \frac{52r}{4} + \frac{24}{4} = 0$

$6r^2 - 13r + 6 = 0$

$(3r - 2)(2r - 3) = 0$

$r = 2/3 \quad r = 3/2$

$\therefore a = \frac{20}{3(1+r)} \rightarrow \boxed{a_1 = 4 \quad a_2 = 8/3}$ ← final answer

97. Solve for x: $$\sum_{k=1}^{2} 4(x)^{k-2} + \sum_{k=1}^{\infty} -4(x)^{k-1} = 10$$

$$\underbrace{4(x^{-1})}_{\frac{4}{x}} + \underbrace{4x^{0}}_{4} + \left[-4 + -4x^{1} + -4x^{0} + \dots\right] = 10$$

this is an infinte series, where $a = -4$ and $r = x$

So $S = \dfrac{-4}{1-x}$

$$\frac{4}{x} + 4 - \frac{4}{1-x} = 10 \leftarrow \text{mult by } x(1-x)$$

$$4(1-x) + 4x(1-x) - 4x = 10x(1-x)$$

$$4 - 4x + 4x - 4x^2 - 4x = 10x - 10x^2$$

$$-4x^2 - 4x + 4 = 10x - 10x^2$$

$$6x^2 - 14x + 4 = 0$$

$$(6x - 2)(x - 2) = 0$$

$$\boxed{x = 2/6 = 1/3} \quad \text{or } x = \cancel{2} \leftarrow \text{reject, since } r < |1|$$

Wow, that was pretty tough... but cool!

98. a) State all restrictions on the value of x if $\dfrac{2\log x}{3+\log x}=\dfrac{-2\log x}{3-\log x}$

$x > 0$

$\log x \neq 3 \rightarrow x \neq 10^{3}$

or $x \neq 1000$

$\log x \neq -3 \rightarrow x \neq 10^{-3}$

or $x \neq \frac{1}{1000}$

b) Solve the above equation for x. cross mult!

$(2\log x)(3-\log x) = (-2\log x)(3+\log x)$ let $A = \log x$

$2A(3-A) = -2A(3+A)$

$6A - 6A^2 = -6A - 2A^2$

$-4A^2 + 12A = 0$

$-4A(A-3) = 0$

$A = 0$ or $A = 3$

$\log x = 0$ $\log x = 3$

$x = 1$ $x = \cancel{1000}$ reject

99. Solve for x: $8^{\log_2 x} - 25^{\log_5 x} = 4x - 4$

$2^{3\log_2 x} - 5^{2\log_5 x} = 4x - 4$

$2^{\log_2 x^3} - 5^{\log_5 x^2} = 4x - 4$

$x^3 - x^2 = 4x - 4 \leftarrow$ solve using your graphing calc, or use long division from Math 11

$x = \cancel{-2}, 1, 2$ reject

100. Solve for x: $\log_3 x + \log_9 x = 6.5$, accurate to 2 decimal places.

$$\log_3 x + \frac{\log_3 x}{\log_3 9} = 6.5$$

$$\log_3 x + \frac{\log_3 x}{2} = 6.5 \quad \text{mult. by } 2$$

$$2\log_3 x + \log_3 x = 13$$

$$3\log_3 x = 13$$

$$\log_3 x = \frac{13}{3} \longrightarrow x = 3^{13/3} = 116.82$$

Answers:

1. c
2. d
3. b
4. d
5. a
6. b
7. c
8. (.3869, 1.5296)
9. b
10. 0.27
11. c
12. b
13. c
14. a
15. b
16. a
17. a
18. d
19. 20
20. c
21. c
22. b
23. a
24. b
25. a
26. a
27. 0.18
28. a
29. b
30. a
31. c
32. a
33. b
34. a
35. d
36. c) x= 3.17
37. d
38. c
39. d
40. c
41. d
42. a (this one is tricky!)
43. b
44. b
45. c
46. c
47. 11
48. a) 2 b) 1 c) 4 d) 3
49. c
50. a
51. d
52. a
53. b
54. c
55. $A = A_0 e^{-0.288t}$
56. c
57. d
58. x=6 (reject -2)
59. b
60. d
61. a
62. There is a typo on this question. It should read $f(x)=2^{-x}$, and the answer is b.
63. c
64. b
65. b
66. d
67. t=13.2 years
68. d
69. c
70. b
71. d
72. d
73. b
74. c
75. x= 1/2 (reject -3)
76. d
77. d
78. d
79. b
80. k=0.0127
81. c
82. c
83. b
84. c
85. b
86. b
87. x=-1 (reject 6)
88. -8, 12, -18 or -18, 12, -8
89. x= 16 or x = 1/4
90. a) $\frac{50}{\sqrt{2}}$ b) 682.84
91. $f^{-1}(x) = 3^{\frac{x-1}{2}}$
92. a $\pm\sqrt{\frac{1}{2}}$) b) 32
93. -81 or 16.2 (hint: r= $\pm\frac{2}{3}$
94. x= $1, 2, \sqrt{2}, \frac{1}{2\sqrt{2}}$
95. 6.63 years
96. a=4 or a=8/3 (hint: set up a system, and use substitution!)
97. x=1/3 (reject 2)
98. a) $x>0$, $x \neq .001$, $x \neq 1000$ b) x=1
99. x=1,2
100. x=116.82