Principles of Math 12 Name:

Combinatorics Review

(pm30_june01_de.pdf)
1.

Use the following information to answer the next question.

The following instructions were given on a survey.

> Place an **X** in the box beside the activities that interest you when you are on vacation. You may place an **X** in as many boxes as you like, or you may leave all boxes blank.
>
> ☐ sightseeing
> ☐ theatre
> ☐ hiking
> ☐ skiing
> ☐ museums
> ☐ golfing
> ☐ shopping

Before the results of all the completed surveys can be tabulated, the number of different possible combinations that can be selected must be determined.

What is the number of different possible combinations?

$_7C_0 + {_7C_1} + {_7C_2} + {_7C_3} + {_7C_4} + {_7C_5} + {_7C_6} + {_7C_7}$

A. 28
B. 128
C. 5 040
D. 13 700

2.

A school committee consists of 1 vice-principal, 2 teachers and 3 students. The number of different committees that can be selected from 2 vice-principals, 5 teachers, and 9 students is

1 2 3

| 2 | 5 | 9 |

V T S

A. 20 160

B. 8 008

C. 1 680

D. 90

$_2C_1 \times {_5C_2} \times {_9C_3}$

3.

Tim and Rebecca are the first and second students in a line of 7 students waiting to buy tickets for a concert. The number of different orders in which the remainder of the students can line up behind them is

So, only 5 students can shuffle

$\underset{\text{Tim}}{\underline{1}} \times \underset{\text{Reb}}{\underline{1}} \times \underline{5} \times \underline{4} \times \underline{3} \times \underline{2} \times \underline{1} = 5!$

A. 5!

B. 7!

C. $5! \times 2!$

D. $\frac{7!}{2!}$

4.

At one time, a standard licence plate consisted of any 2 consonants followed by any 4 digits. Later, the standard licence plate was changed to consist of any 3 consonants followed by any 3 digits. Given that all 20 consonants can be used, and that any consonant and any digit can be repeated, how many **more** standard licence plates were available after this change?

A. 3 009 600

B. 4 000 000

C. 8 000 000

D. 12 000 000

originally: $\underset{\text{con}}{\underline{20}}\ \underset{\text{con}}{\underline{20}}\ \underset{\#}{\underline{10}}\ \underset{\#}{\underline{10}}\ \underset{\#}{\underline{10}}\ \underset{\#}{\underline{10}} = 4{,}000{,}000$

later: $\underset{\text{con}}{\underline{20}}\ \underset{\text{con}}{\underline{20}}\ \underset{\text{con}}{\underline{20}}\ \underset{\#}{\underline{10}}\ \underset{\#}{\underline{10}}\ \underset{\#}{\underline{10}} = 8{,}000{,}000$

$8{,}000{,}000 - 4{,}000{,}000 = 4{,}000{,}000$

5.

In a particular town, all of the streets run north–south or east–west. A student lives 5 blocks west and 3 blocks south of a school. The number of different routes, 8 blocks in length, that the student can take to get to the school is __________.

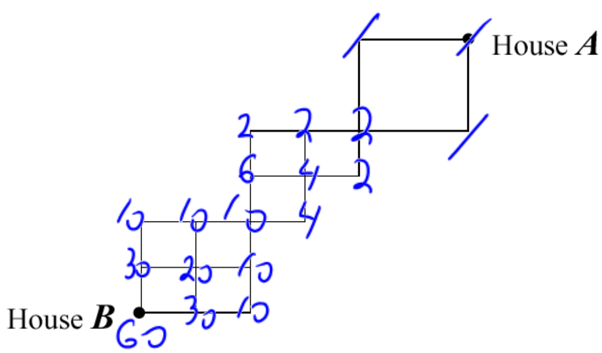
$$\frac{8!}{5!\,3!} = 56$$

(pm30_jan02_de.pdf)

6.

Use the following information to answer the next question.

A paperboy who delivers papers on his bike can travel only on the trails represented in the diagram below.



The number of different trails that the paperboy can take to get from house *A* to house *B* without backtracking is

A. 13

B. 32

C. 60

D. 72

7.

The number of different arrangements of 3 boys and 4 girls in a row, if the girls **must** stand together, is represented by

A. $4! \times 4!$

B. $3! \times 4!$

C. $4! \times 4! \times 2!$

D. $3! \times 4! \times 2!$

$\boxed{B_1}$ $\boxed{B_2}$ $\boxed{B_3}$ $\boxed{G_1 G_2 G_3 G_4}$

4! ways of mixing this up

4 ways of positioning the girls

3! ways of mixing up boys

$= 4 \times 3! \times 4! = 4!\,4!$

8.

The students in a music department have practised 6 contemporary and 5 traditional choruses. For their concert, they will choose a program in which they present 4 of the contemporary and 3 of the traditional choruses. How many different programs can be presented, if the order of the choruses does **not** matter? ← combination

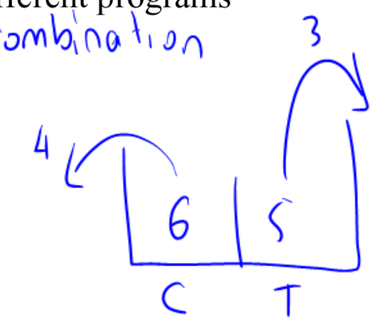
A. 25

B. 35

C. 150

D. 330

$_6C_4 \times {}_5C_3$



9.

All telephone numbers are preceded by a 3-digit area code. In the original Bell Telephone System of assigning area codes, the first digit could be any number from 2 to 9, the second digit was either 0 or 1, and the third digit could be any number except 0. In this system, the number of different area codes possible was

A. 126

B. 144

C. 160

D. 576

$\frac{8}{2-9} \times \frac{2}{0 \text{ or } 1} \times \frac{9}{1-9}$

10.

A term of the binomial expansion $(ax + y)^8$, where $a > 0$, is $112x^2y^6$. The value of a, correct to the nearest whole number, is ___2___.

this is term 7

$t_{k+1} = {}_nC_k\, a^{n-k}\, b^k$

$k = 6 \qquad b = y$

$a = ax \qquad n = 8$

$t_7 = {}_8C_6\,(ax)^2\, y^6$

$112x^2y^6 = 28a^2x^2y^6 \longrightarrow$

$112 = 28a^2$

$4 = a^2$

$\underline{a = 2}$

(pm30_jan01_de.pdf)

11.

If all of the letters in the word **DIPLOMA** are used, then the number of different 7-letter arrangements that can be made beginning with 3 vowels is

$\boxed{IOA}$ $3!$

$\underline{4} \times \underline{3} \times \underline{2} \times \underline{1}$

$= 3! \times 4!$

A. 24
B. 144
C. 720
D. 5 040

12.

Use the following information to answer the next question.

At a company picnic, employees used their cooking skills in a chili cook-off. Employees were required to select the specified number of items from each of the following lists.

$_2C_1$ **List A** (select 1)	$_3C_2$ **List B** (select 2)	$_4C_3$ **List C** (select 3)	$_3C_2$ **List D** (select 2)	$_4C_2$ **List E** (select 2)
• 250 g pork • 250 g beef	• 375 mL tomato sauce • 375 mL salsa • 375 mL mixture of water and ketchup	• 1/2 cup onion • 1/4 cup green pepper • 1/4 cup red pepper • 1 cup mushrooms	• 375 mL kidney beans • 375 mL pork and beans • 375 mL lima beans	• 1 tbsp seasoning salt • 1 tbsp garlic salt • 2 tbsp chili powder • 2 tbsp hot sauce

The number of different chili recipes that were possible was

$_2C_1 \times {_3C_2} \times {_4C_3} \times {_3C_2} \times {_4C_2}$

A. 18
B. 120
C. 144
D. 432

term 7

13.

If one term in the expansion of $(x-b)^{10}$, $b>0$, is $\frac{76\,545}{32}x^4$, then the value of b, correct to the nearest tenth, is __________.

$t_{k+1} = {}_nC_k\, a^{n-k}\, b^k$ $\quad k=6 \quad n=10 \quad a=x \quad b=-b$

$\frac{76545}{32} x^4 = {}_{10}C_6\, x^4 (-b)^6$

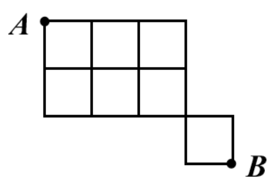
$\frac{76545}{32} = 210(b)^6$

$b^6 = \frac{76545}{6720} \rightarrow b = \sqrt[6]{\frac{76545}{6720}} = 3/2 = 1.5$

14.

Use the following information to answer the next question.

A student must draw a path from point A to point B in the diagram below.



If each path must be drawn along the lines such that it is always getting closer to B, then the number of paths that the student can draw is

A. 12
B. 20
C. 40
D. 70

$\frac{5!}{3!2!} \times \frac{2!}{1!1!}$ or ${}_5C_3 \times {}_2C_1 =$

15.

Use the following information to answer the next question.

Library books at Grande Prairie Regional College all have bar codes. The bar codes have 14 digits, and the first 8 digits are always 3 1847 000. The remaining 6 spaces can be filled by any digits. An example is shown below.

3 1847 000 565162

The number of different bar codes available for books at this library is

A. 6!

B. 10!

C. 10^6

D. ${}_{10}P_6$

$\frac{10}{\#}\ \frac{10}{\#}\ \frac{10}{\#}\ \frac{10}{\#}\ \frac{10}{\#}\ \frac{10}{\#}$

(m30june_2000.pdf)

16.

3rd term

In the expansion of $(2x - 1)^{10}$, the coefficient of the term containing x^8 is

$t_{k+1} = {}_nC_k\, a^{n-k}\, b^k$

$k = 2 \quad n = 10$

$a = 2x$

$b = -1$

$t_3 = {}_{10}C_2 (2x)^8 (-1)^2$

$= 45(256)x^8(1)$

A. –11 520

B. 45

C. 256

D. 11 520

17.

Five different outfits are to be displayed in a store window. From 5 different winter outfits, 3 must be chosen, and from 4 different fall outfits, 2 must be chosen. These outfits will then be arranged in a row in the store window. The number of displays that can be made by choosing the outfits and then arranging them in the window is

3 | 5 | 4 | 2

W Fall

$_5C_3 \times {}_4C_2 = 60$ ways to choose 5 outfits

A. 300

B. 3 600

C. 7 200

D. 86 400

Now arrange them in $5!$ ways $\therefore 5! \times 60$

18.

A set of 3 different Mathematics books, 4 different Physics books, and 2 different English books are arranged on a shelf. If the books on each subject are to be kept together, then the number of different arrangements possible for the books is

$\boxed{M_1\ M_2\ M_3}$ $\boxed{P_1\ P_2\ P_3\ P_4}$ $\boxed{E_1\ E_2}$

$3!$ $4!$ $2!$

A. 288

B. 864

C. 1 260

D. 1 728

$3! \times 4! \times 2! \times 3!$ ways to mix up the 3 groups

19.

A dance class has 14 students. From these, 3 dancers are to be chosen to do a demonstration. The number of different groups of 3 that can be formed is __364__.

$_{14}C_3$

20.

Codes with 4 digits are to be made from the digits 1, 2, 3, 4, 5, 6, and 7. If repetitions are **not** permitted and each code must contain 2 odd digits followed by 2 even digits, then the number of codes that can be formed is

A. 72 codes

B. 144 codes

C. 210 codes

D. 840 codes

$\frac{4}{odd_1}\ \frac{3}{odd_2}\ \frac{3}{even_1}\ \frac{2}{even_2}$

(m30de991.pdf)

21.

The number of different arrangements of all the letters in the word **TOOTH** is

A. $5!$

B. $\frac{5!}{2!}$

C. $\frac{5!}{2!2!}$

D. $\frac{5!}{3!2!}$

$\frac{5!}{2!\,2!}$

22.

The names of 15 students are put into a hat. Of these, 4 students are going to be chosen for a school trip to Saskatoon. The number of different possible groups of students is

A. 4!

B. 15!

C. 1 365

D. 32 760

$_{15}C_4$

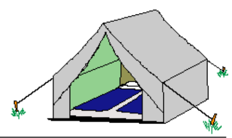
23.

Use the following information to answer the next question.

The manager of a sports store wants 5 different tents to be illustrated on one page of a sales flyer. The illustrations will be positioned one above the other.

Dome tent

Scout tent



Rain tent

Hiking tent

Spacious tent

In the flyer, the Dome, Rain, and Spacious tents are blue, the Scout tent is beige, and the Hiking tent is yellow. The blue tents can appear in any order; however, the manager does not want to have one blue tent immediately after another blue tent. If this is the only restriction, then how many different positions are possible?

A. 2
B. 12
C. 20
D. 60

$\boxed{\text{D, R, Sp}}$ Blue $\quad \boxed{\text{Sc}}$ Beige $\quad \boxed{\text{H}}$ yellow

3! ways of arranging blue tents

$$\frac{3}{\text{blue}} \times \frac{2}{} \times \frac{2}{\text{blue}} \times \frac{1}{} \times \frac{1}{\text{blue}}$$

4th term

24.

In the expansion of $(a+b)^{10}$, the numerical coefficient of a^7b^3 is __________.

$t_{k+1} = {}_nC_k\, a^{n-k} b^k$ $\quad n=10 \quad k=3$

$t_4 = {}_{10}C_3\, a^7 b^3$

${}_{10}C_3 = \boxed{120}$

25.

The number of 3-digit numbers **less than 400** that can be formed if the last digit is either 4 or 5 is __________.

(m30de2000.pdf)

$\underline{3} \times \underline{10} \times \underset{4 \text{ or } 5}{\underline{2}} = 60$

26.

The value of ${}_aC_5,$ where $a \in N$ and $a > 5,$ is equal to the value of

A. ${}_5C_a$ ie ${}_{10}C_6 = {}_{10}C_4$ or ${}_{13}C_{11} = {}_{13}C_2$, etc

B. ${}_{a-5}C_5$

C. ${}_aC_{a-5}$

D. ${}_aC_{5-a}$

27.

Assume that in a given set of 3-digit area codes, the middle digit of each code is either "0" or "1." Which of the following conditions on the digits would result in exactly 180 area codes?

A. The first digit cannot be zero, and the third digit must be different from the first. $\underline{9} \times \underline{2} \times \underline{9}$

B. There are no restrictions on the possible values. $\underline{10} \times \underline{2} \times \underline{10}$

C. All three digits must be different. $\underline{9} \times \underline{2} \times \underline{8}$

D. The first digit cannot be zero.

$\underline{9} \times \underline{2} \times \underline{10}$

28.

If $(x-1)^5 = a_5x^5 + a_4x^4 + a_3x^3 + a_2x^2 + a_1x + a_0$, then the value of $a_5 + a_4 + a_3 + a_2 + a_1 + a_0$ is

$${}_5C_0\, x^5 + {}_5C_1\, x^4(-1) + {}_5C_2\, x^3(-1)^2 + {}_5C_3\, x^2(-1)^3 + {}_5C_4\, x(-1)^4 + {}_5C_5\, (-1)^5$$

A. –32
B. 0
C. 1
D. 32

$$= 1x^5 - 5x^4 + 10x^3 - 10x^2 + 5x - 1$$

$$1 - 5 + 10 - 10 + 5 - 1 = 0 \checkmark$$

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29.

For a certain college program, a student must take an English course. The student must also take any 4 of the following courses: Mathematics, Chemistry, Physics, Psychology, Biology, and French. The number of possible 5-course programs that a student could take to complete this program is

1 4

1 | 6

English others

A. ${}_6C_4$
B. ${}_7C_5$
C. ${}_6P_4$
D. ${}_7P_5$

$${}_1C_1 \times {}_6C_4$$

$$= 1 \times {}_6C_4 = {}_6C_4$$

30.

If ${}_nP_r = 3\,024$ and ${}_nC_r = 126$, then the value of r is

A. 4

B. 6

C. 9

D. 24

$$\boxed{\frac{n!}{(n-r)!}} = 3024 \qquad \boxed{\frac{n!}{(n-r)!}}\frac{}{r!} = 126 \quad r! = \frac{3024}{126}$$

$$\frac{3024}{r!} = 126 \qquad r! = 24 \qquad \boxed{r = 4}$$

31.
What is the 8th term of the 14th row of Pascal's Triangle?
Ans:______________

$${}_{13}C_7 = 1716$$

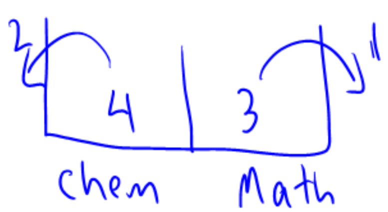
32.
What is the middle term of the expansion of $(3x-2y)^8$? Ans:______________ t_5

$$t_{k+1} = {}_nC_k \, a^{n-k} \, b^k \qquad k = 4 \quad n = 8$$

$$t_5 = {}_8C_4 \, (3x)^4 (-2y)^4$$

$$= 70 \cdot 3^4 \cdot x^4 \cdot (-2)^4 \cdot y^4 = 90720\, x^4 y^4$$

33. How many different committees of two chemists and one mathematician can be formed from the four chemists and three mathematicians on the faculty of a small college?



$${}_4C_2 \times {}_3C_1 = 18$$

34. A college team plays 10 football games during the season. In how many different ways can it end the season with 5 wins, 4 losses, and a tie?

WWWWW LLLL T

$$= \frac{10!}{5!\,4!} = 1260$$

35. A shipment of 10 television sets includes exactly three that are defective. In how many ways can a hotel purchase four of these sets and receive at least two that are defective?

2 | 3 | 7 | 2

Def   Non-Def.

$\underline{2}$ or $\underline{3}$

$${}_3C_2 \times {}_7C_2 + {}_3C_3 \times {}_7C_1 = 70$$

36. How many different bridge hands are possible containing 5 spades, 3 diamonds, 3 clubs, and 2 hearts?

$${}_{13}C_5 \times {}_{13}C_3 \times {}_{13}C_3 \times {}_{13}C_2 = 8{,}211{,}173{,}256$$

37. Solve: $\dfrac{(n+1)!}{n!} = 9$

$$\frac{(n+1)\cancel{(n)!}}{\cancel{n!}} = 9$$

$$n+1 = 9$$

$$n = 8$$

38. Solve: $\dfrac{n!}{(n-2)!} = 20$

$$\frac{n(n-1)\cancel{(n-2)!}}{\cancel{(n-2)!}} = 20$$

$$n(n-1) = 20$$

$$n^2 - n = 20$$

$$n^2 - n - 20 = 0$$

$$(n-5)(n+4) = 0$$

$$n = 5, \cancel{-4}$$

39. Solve: $\frac{3(n+1)!}{(n-1)!}=126$

$$\frac{3(n+1)(n)\cancel{(n-1)!}}{\cancel{(n-1)!}}=126$$

$$3n(n+1)=126$$

$$\frac{3n^2}{3}+\frac{3n}{3}-\frac{126}{3}=\frac{0}{3}$$

$$n^2+n-42=0$$

$$(n+7)(n-6)=0$$

$$n=\cancel{-7},\ 6$$

40. Solve: $\dfrac{2n!}{(n-3)!} = 84n$

$$\frac{2\cancel{n}(n-1)(n-2)\cancel{(n-3)!}}{\cancel{(n-3)!}} = 84\cancel{n}$$

$$\frac{2}{2}(n-1)(n-2) = \frac{84}{2}$$

$$(n-1)(n-2) = 42$$

$$n^2 - 3n + 2 = 42$$

$$n^2 - 3n - 40 = 0$$

$$(n-8)(n+5) = 0$$

$$n = 8, \cancel{-5}$$

41. How many three digit numbers can be made up from the digits 1 to 5 if: a) no repetition is allowed and the number must be greater than 500? b) repetition is allowed and the number must be less than 300?

a) $\underline{\ 1\ }\ \underline{\ 4\ }\ \underline{\ 3\ } = 12$

5

b) $\underline{\ 2\ }\ \underline{\ 5\ }\ \underline{\ 5\ } = 50$

2 or 1

42. Show algebraically that ${}_nP_n = {}_nP_r \times {}_{(n-r)}P_{(n-r)}$ for all n>r.

${}_nP_n$	${}_nP_r \times {}_{(n-r)}P_{(n-r)}$
$= \dfrac{n!}{(n-n)!}$	$\dfrac{n!}{(n-r)!} \times \dfrac{(n-r)!}{(n-r-(n-r))!}$
$= \dfrac{n!}{0!}$	$= \dfrac{n!}{\cancel{(n-r)!}} \times \dfrac{\cancel{(n-r)!}}{0!}$
$= n!$	$= n!$ Q.E.D.

43. a) How many arrangements are there of the word BASKETBALL? b) How many of these arrangements begin with K? c) How many of the arrangements start with a B? d) In how many arrangements would the two L's be together?

a) $\frac{10!}{2!\,2!\,2!} = 453,600$

b) $\underset{1}{\boxed{K}} \times \frac{9!}{2!\,2!\,2!} = 45,360$

c) omit

d) omit

44. Solve: ${}_{(n+1)}C_3 = {}_nC_2$

$$\frac{(n+1)!}{(n+1-3)!\,3!} = \frac{n!}{(n-2)!\,2!}$$

$$\frac{(n+1)!}{\cancel{(n-2)!}\,(6)} = \frac{n!}{\cancel{(n-2)!}\,(2)} \leftarrow \text{cross multiply}$$

$$\frac{(n+1)!}{n!} = \frac{6}{2}$$

$$\frac{(n+1)\cancel{(n)!}}{\cancel{n!}} = 3$$

$$n+1 = 3$$

$$\boxed{n=2}$$

45. Show that $\frac{1}{(a-1)!n!}+\frac{1}{a!(n-1)!}=\frac{(a+n)}{a!n!}$

write LHS as 1 fraction!

$$\frac{a!(n-1)!}{a!(n-1)!}\cdot\frac{1}{(a-1)!\,n!}+\frac{1}{a!\,(n-1)!}\cdot\frac{(a-1)!\,n!}{(a-1)!\,n!}$$

$$=\frac{a!\,(n-1)!+n!\,(a-1)!}{a!\,(n-1)!\,(a-1)!\,n!},$$

and $a!=a(a-1)!$

$n!=n(n-1)!$

$$=\frac{a(a-1)!\,(n-1)!+n(n-1)!\,(a-1)!}{a!\,(n-1)!\,(a-1)!\,n!}$$

← now, factor GCF

$$=\frac{\cancel{(a-1)!}\,\cancel{(n-1)!}\,[a+n]}{a!\,\cancel{(n-1)!}\,\cancel{(a-1)!}\,n!}$$

$$=\frac{a+n}{a!\,n!}$$

Q.E.D. Wow, pretty tough!

Answers:

1. b
2. c
3. a
4. b
5. 56
6. c
7. a
8. c
9. b
10. 2
11. b
12. d
13. 1.5
14. b
15. c
16. d
17. c
18. d
19. 364
20. a
21. c
22. c
23. b
24. 120
25. 60
26. c
27. d
28. b
29. a
30. a
31. ${}_{13}C_7=1716$
32. $90720x^4y^4$
33. 18
34. 1260
35. 70
36. 8,211,173,256
37. 8
38. 5
39. 6
40. 8
41. a)12 b) 50
42. see solution sheet
43. a) 453600 b) 45360 c) 90720 d) 90720
44. 2
45. see solution sheet

Note to teachers:

The questions here come from a variety of sources. Most come from Alberta provincial exams, or are based on questions from those documents. A few came from some of my university texts.

I generally hand this out at the beginning of the unit (including the answer key), and I collect it the day of the test. I flip through the booklet just to see if there is writing on each page, and I give the students a few marks. During the unit, I have a few photocopied solution manuals (showing all my steps) floating around the class as well. Students can sign them out and take them home if they wish.

If you find any errors in the answer key, or have any suggestions that I could add, feel free to email me at kdueck@sd42.ca and I'll be happy to reply.

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PS Thanks to Gretchen McConnell for helping with error checking my answer keys! It's been much appreciated!